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Failure Predictions in Repairable Multi-Component Systems.

Woodrow Thomas Roberts Jr
Louisiana State University and Agricultural & Mechanical College

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Roberts, Woodrow Thomas, Jr., Ph.D.

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U·M·I
300 N. Zeeb Rd.
Ann Arbor, MI 48106

FAILURE PREDICTIONS IN REPAIRABLE MULTI-COMPONENT SYSTEMS

A Dissertation

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by
Woodrow T. Roberts, Jr.
B.S., Auburn University, 1966
M.B.A., Louisiana State University, 1974
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LIST OF SYMBOLS

β	Shape parameter of intensity function for non-homogenous Poisson Process
$\hat{\beta}$	Maximum likelihood estimate of β
β_W	Shape parameter or slope of the Weibull curve
λ	Failure rate intensity of a homogenous Poisson Process
$\hat{\lambda}$	Maximum likelihood estimate of λ
η	Characteristic life - life of 63.2% of the distribution of the failure data for a component, scale parameter of Weibull distribution
Σ	Summation
CHR	Constant hazard rates
d	Duration of mission
e	Exponential
$E[N(t)]$	The expected number of failures of a system during its age$(0,t)$
$f(t)$	The absolute probability density function
$F(t)$	Distribution function of the time-to-failure random value, t (fraction failing)
$f(t)dt$	The proportion of a population of devices starting at time,$t=0$, which fail in the time interval $(t,t+dt)$
$f(x)$	Poisson distribution function
HPP	Homogenous Poisson Process
$h(t)$	The hazard function of a probability distribution of time-to-failure

LIST OF SYMBOLS (continued)

$h(t)dt$	The proportion of a population of devices which have not failed prior to time, t, but which do fail in the interval $(t,t+dt)$
IHR	Increasing hazard rates
IIED	Independently and identically exponentially distributed
k	Number of copies of the system
ln	Natural logarithm
MLE	Maximum likelihood estimate
n	Number of events or samples
NHPP	Non-homogenous Poisson Process
N_q	Number of failures experienced by the q^{th} system
$N(T)$	The number of failures experienced by the system during time, T
PHM	Proportional hazards modelling
q	The specific system copy, $(q=1...k)$
ROCOF	Rate of occurrence of failures of a system
$R(t)$	Mission reliability
S	Sample standard deviation
S_q	Starting time for system q
$S_{\bar{x}}$	Standard deviation of the mean
t	Students t value
t	Age of the system
t_o	Starting point or origin of distribution

LIST OF SYMBOLS (continued)

T_q	Time of the last failure if failure is truncated, i.e., $X_{N_q,q}=T_q$ or the ending time of the system observation period if t truncated, i.e., $X_{N_q,q}<T_q$
U	Centroid test statistic
$u(t)$	Non-homogenous Poisson Process failure intensity function
X_{iq}	Age of the system at the *ith* failure, in hours $(i=1\ldots q)$, $(q=1\ldots k)$
$x_1,x_2\ldots n$	Values of independent variables
$X_1,X_2\ldots X_n$	Interarrival values, the intervals between successive events

ABSTRACT

The subject of this research is the prediction of failures in repairable multi-component systems from statistical models that utilize the historical failure data for the systems. Failures occurring in repairable systems are examples of a series of discrete events which occur randomly in a continuum. Such stochastic point processes are analyzed using the statistics of event series. The Crow nonhomogenous Poisson process, NHPP, model is recognized by the reliability community as being one of the best models for repairable systems.

The objective of this research is to show that the Crow NHPP model, with its overall failure predictions for a repairable system, can be utilized as a guide for testing the accuracy of a Monte Carlo simulation that utilizes the individual component Weibull distribution parameters to predict system failures.

Failure data, from multiple versions of six different mechanical systems, are modelled by Crow's NHPP model. A program is presented that performs an iteration of Crow's equations to obtain the NHPP parameters that are then utilized to develop a failure intensity function for each respective system. Failure predictions are then determined from the mean value function of the NHPP model.

The individual component failure data for each system are fitted to Weibull distributions and the resulting distribution function parameters are utilized in the respective Monte Carlo simulations.

In each of the six cases a Monte Carlo simulation, based on the Weibull distributions of the major component failure modes, is used to predict the number of failures expected for each system.

The Monte Carlo simulation predictions are shown to closely match the Crow nonhomogenous Poisson process predictions for the respective systems. In addition, the Monte Carlo simulations give failure prediction results that can be traced to indi-

vidual components. The Crow model predicts when the overall system will be down, and then the simulation predicts the number of failures from each of the included components.

The simulation can identify a finite number of parts that contribute to the overall system downtime. This information can be used to design an optimum preventive maintenance program or guide research into more reliable components.

CHAPTER 1

INTRODUCTION

Identification of the reasons for failures of process plant machines is the first step in obtaining increased reliability. The only satisfactory arbiter of reliability is performance in the field. If the user is to help himself, there is no alternative to the adequate collection of failure records. In order to get increased reliability, the causes of failure must be identified and appropriate actions taken to eliminate or reduce them. (45)

Reliability is defined as the probability that a component, device, or system will perform satisfactorily for the designated period of time under design conditions. The concept of reliability as a probability means that any attempt to quantify it must involve the use of statistical methods. Whether an item works for a particular period is a question which can be answered as a probability. (40)

Reliability engineering is a predictive, probabilistic, applied science, and the collection and analysis of past operating experience is used to predict and in some cases shape, future events. (46)

In general, the most accurate reliability predictions are those based on actual field experience with similar equipment in the specific application. To obtain high quality reliability data, the collection and analysis of failure statistics of each component and equipment type from a representative population operating under identical conditions is needed. This is obviously an ideal situation and never possible in practice for mechanical equipment.

Where a computer system exists for maintenance planning, there exists the basis for a comprehensive in-house reliability database. Fairly recent developments involve the use of a microcomputer and database management system as an add-on facility to existing computerized maintenance schemes.

Companies in the chemical process industry have only now started to look at incorporating reliability engineering into the design, construction, operation, and maintenance of their plants. These techniques come from the aerospace industries and more recently the power generation or utility industries. Florida Power and Light, for example, recently was the first United States company to win the prestigious Deming Award as a result of improvements in availability, reliability, and quality of service. Florida Power and Light has been utilizing reliability techniques since the mid 1980's and credits their overall improvement at least partially from the use of this technology. (41)

The subject of this research is the statistical analysis and creation of models from failure data for components and systems utilized in the chemical process industry. The databases originated at the plastics producing plants in Dow Chemical's Louisiana Division. The failure data bank and the library of distribution parameters that have been generated are unique to the knowledge base of Maintenance Engineering and Reliability Engineering as applied to the Chemical Process Industry.

The Dow Louisiana Division's Maintenance Management Information System, MMIS, was utilized in conjunction with an IBM PC to develop a failure data bank for mechanical components and systems of components. REFLEX, a database management software system, was utilized to download, filter, sort and categorize the failure data to prepare it for analysis.

The data sample was limited to the following critical mechanical equipment in the plastics producing plants in Dow's Louisiana Division:

1. Reciprocating compressors
2. Reactors and agitator systems
3. Centrifugal compressors

The failure data of the individual components was analyzed statistically. Procedures given in the literature were utilized to fit the most likely statistical distribution to the data for individual component failure modes and determine the values of the distribution parameters.

Most complex systems are repaired, not replaced, when they fail. Recent literature (2) has stressed that the usual nonrepairable reliability models, such as the Weibull distribution, are not appropriate for repairable system reliability analysis and have suggested the use of the nonhomogeneous Poisson process models.

The reliability analyses of a repairable system under customer use involve data generated by multiple systems. Crow (7) proposed the power law nonhomogeneous Poisson process for this type of analysis and developed statistical procedures for maximum likelihood estimation, goodness-of-fit and confidence bounds. The pioneering work involved in the paper presented by Larry Crow at the 1990 Annual Reliability and Maintainability Symposium, RAMS, has been extended in this research by modeling repairable multicomponent mechanical systems.

The approach taken was to analyze and model component failure data where possible and then to use the component distribution parameters to simulate the overall system reliability. Predictions from these models were compared to the hypotheses of the nonhomogeneous Poisson process model.

Multi-component repairable systems cannot be modeled by continuous distributions, such as the Weibull, using time between failures of the system as a whole. O'Connor (40) explains that failures occurring in repairable systems are examples of a series of discrete events which occur randomly in a continuum. These situations, which are stochastic point processes, are analyzed using the statistics of event series. The Crow (7) model or power law nonhomogeneous Poisson process is an exceptionally useful model for repairable systems analysis.

In this research, failure data for multiple versions of certain mechanical systems are modelled by Crow's nonhomogeneous Poisson process, NHPP. The expected number of failures predicted for the respective system by the Crow model is considered the standard for such a system and attempts are made to match that prediction using a Monte Carlo simulation that utilizes Weibull parameters of the major components of the system.

The objective is to prove that a simulation based on Weibull parameters of the major component failure modes is able to duplicate the overall system prediction that the Crow NHPP model gives. This technique of using the Crow NHPP model, which uses overall system data, to serve as a guide for testing the accuracy of a simulation, using Weibull parameters of individual component failure modes, is new and unique.

The simulations, based on fitting component failure data to a continuous distribution such as the Weibull, are more valuable in that they give failure prediction results that can be traced to individual components. The advantage here is that only the major component failure modes will need to be included in the simulation and the NHPP model can be utilized as a gauge to determine when the simulation has the appropriate component failure modes included.

The uniqueness in this research lies in the use of the simulation approach in conjunction with the Crow NHPP model so that failure modes can be better identified. The Crow model predicts when the overall system will be down, and then the simulation predicts the frequency of failures from each of the included components. The simulation can identify a finite number of parts that contribute to the overall system downtime. This information can be used to design an optimum preventive maintenance program or to guide research into more reliable components or parts.

CHAPTER 2

LITERATURE SEARCH

A comprehensive introductory book on reliability concepts is O'Connor's Practical Reliability Engineering. (40) O'Connor explains that engineering education is basically deterministic, and does not usually pay sufficient attention to variability. Yet variability and chance play a vital role in determining the reliability of most products or equipment. He explains that reliability is concerned with failures in the time domain. This distinction marks the difference between traditional quality control and the modern approach to reliability.

Throughout the product life cycle the reliability is assessed, first by initial predictions based upon past experience in order to determine feasibility and set objectives, then by refining the predictions as detail design proceeds and subsequently by recording performance during the test, production and in-use phases. This performance is fed back, hopefully, to generate any necessary corrective action . The feedback also can provide data and guidelines for future products. (40)

Reliability statistics can be broadly divided into the treatment of discrete functions, continuous functions and point processes. Situations where a thing either works or doesn't work are described by discrete functions. In reliability the concern is often with two-state discrete systems, since equipment is in either an operational or a failed state. Continuous functions describe those reliability situations which are governed by a continuous variable, such as time or distance travelled. The statistics of point processes are used in relation to repairable systems, when more than one failure can occur in a time continuum. (40)

Several references exist on how to fit failure data to continuous statistical distributions: Hahn, (18); Lloyd, (30); Mann, (32); Lipson, (29); Billington, (4); Dhillon, (14). The Reliability of Mechanical Systems (46) edited by John Davidson for the Institute of Mechanical Engineers (London) gives practical guidance on the subject as does Abernethy's Weibull Analysis Handbook (1).

According to Lloyd (30), to specify a relevant probability distribution may be a difficult task, as the mathematical form of the distribution would depend on the "mechanism of failure" of the particular device. Fortunately, the author continues, there are a few probability distributions whose applicability is almost universal. Three of special importance to reliability are the exponential, the Weibull and the log normal distributions of time-to-failure.

In The Reliability of Mechanical Systems (46), readers are warned to distinguish between the Hazard Rate Function and the Failure Rate. The term Hazard Rate Function is used to describe the behavior of nonrepairable components which form part of the system. The term Failure Rate implicitly assumes that the time to failure distribution is exponential and is used to describe the behavior of repairable systems.

The concept of hazard functions can be utilized to derive time-to-failure distributions, in particular the exponential and Weibull distributions. The hazard function, $h(t)$, is a measure of the proneness to failure as a function of age. It is the instantaneous failure rate at time, t.

If $f(t)$ is the (absolute) probability density function, then $f(t)dt$ will represent the proportion of a population of devices starting at time $t=0$, which fail in the time interval $(t, t+ dt)$.

The relationship between the hazard function, h(t), and f(t) is shown to be

$$h(t) = \frac{f(t)}{1 - F(t)} \tag{2.1}$$

where

$$f(t) = \int_o^t f(x)dx \tag{2.2}$$

1-F(t) = the reliability at time t and will be denoted by R(t).

F(t) = the distribution function of the time-to-failure random variable, t.

f(t) = the probability density function.

According to Abernethy (1), the Weibull distribution may be defined mathematically, as follows:

$$F(t) = 1 - e^{-[(t-t_o)/\eta]^\beta} \tag{2.3}$$

where

F(t) = fraction failing

t = failure time

t_o = starting point or origin of the distribution

η = characteristic life or scale parameter

β = slope or shape parameter

F(t) thus defines the cumulative fraction of a group of parts which will fail by a time t. Therefore, the fraction of parts which have not failed up to time t is 1 - F(t). This is called reliability at time t and is denoted by R(t). By rearranging the distribution function, the following can be noted:

$$1 - F(t) = e^{-[(t-t_o)/\eta]^\beta} \tag{2.4}$$

Let $t_o = 0$,

Then,

$$1 - F(t) = e^{-(t/\eta)^{\beta}} \tag{2.5}$$

Which yields,

$$\frac{1}{1 - F(t)} = e^{(t/\eta)^{\beta}} \quad . \tag{2.6}$$

Taking the natural logarithm gives

$$\ln\left(\frac{1}{1 - F(t)}\right) = \left(\frac{t}{\eta}\right)^{\beta} \quad . \tag{2.7}$$

Taking the log again results in

$$\ln \ln\left(\frac{1}{1 - F(t)}\right) = \beta \ln t - \beta \ln \eta \quad . \tag{2.8}$$

Abernethy (1) then shows that this is in the standard form of a straight line:

$$Y = BX + A \tag{2.9}$$

He does this to explain how a Weibull plot can yield a straight line when plotted on Weibull paper. He explains that by choosing ln t as X, the scale on the abscissa and ln ln [1 / 1 - F(t)] as y, the scale on the ordinate, the cumulative Weibull distribution can be represented as a straight line. The log log reciprocal ordinate (Y) scale, represents cumulative probability of failure (or failure percentage), and the abscissa (X) scale is a log scale representing the life value. (1)

Weibull paper can be constructed with common units for the abscissa and ordinate and the slope of the straight line will be β, the shape parameter of the distribution.

Abernethy (1) describes how "Median Ranks" are used to establish F(t) plotting positions. Each failure in a group of tested units will have a certain percentage of the total population failing before it. The first step in establishing a Weibull plot is to order the data from low time to high time failure. The "Median Ranks" are obtained from the tables for the respective sample sizes and the rank order of the time-to-failure.

Formal methods of rank regression and maximum likelihood can then be used to establish the fit. One needs three parameters to describe a Weibull distribution when discussing or reproducing the curve. One is β, the slope or shape parameter, the second is t_0, the location parameter, and the third is η, the scale parameter or characteristic life. The location parameter, t_0, is also called the failure free time or minimum life. When $t_0=0$, the resulting expression is a two-parameter Weibull distribution.

The unique feature of the characteristic life, η, is that it occurs at the 63.2 percent point regardless of the Weibull distribution (i.e., slope). When t is equal to η, β can be disregarded in that then F(t) is always 63.2 percent:

$$
\begin{aligned}
F(t) &= 1 - e^{[-t/\eta]^{\beta}} \\
&= 1 - e^{(-1)^{\beta}} \\
&= 1 - 0.368 = 0.632
\end{aligned}
$$

In the paper by Jardine, et al (25), the statistical analysis of locomotive engine failure data is undertaken. The authors concentrated on verifying the Weibull form of the hazard function. Whether the hazard function approximates a Weibull can be verified by use of a Weibull hazard plot. The degree to which the plot is linear is the degree to which the distribution is truly Weibull.

Summers-Smith (45) gives examples in his work of how Weibull analysis of the service life, obtained from maintenance records, requires a minimum of data and provides insight into failure mechanisms. He explains that β takes on different values depending on whether the incidence of failures is decreasing, remaining constant, or increasing with time. The author defines the following values of β as representative of the following types of failure:

infant mortality, $\beta = 0.5$

random failure $\beta = 1$

wear-out $\beta > 3.4$

Summers-Smith (45) explains that the power of the Weibull analysis is such that it is possible to obtain useful guidance with as few as five failures, and thus the fact that it is restricted to single components is no severe limitation.

The Usher, et al (47), work addresses the topic of "masked" system life data. Life data from systems of components are analyzed to estimate the reliability of the individual components. The exact component causing system failure may be unknown or "masked". That is, the cause may be isolated to some subset of the system's components. The authors (47) present an iterative approach for obtaining component reliability estimates from such data for series systems. The approach is analogous to traditional probability plotting. That is, it involves the fitting of a parametric reliability function to a set of non-parametric reliability estimates (plotting points). The major advantage of the approach is its ability to yield good estimates with much less computation than the method of maximum likelihood.

In the work by Leitao, et al (28), proportional hazards modelling, PHM, is introduced as a non-parametric technique based on the assumption of a loglinear hazard function which can be applied to assess the effect of observed factors on reliability. Ascher and Feingold (2) also analyzed systems repair data and pointed out a need for data analysis methods for repair data. They generally employ parametric models, estimates and confidence limits, based on the Poisson process, the simplest and best known parametric model.

The work by De La Mare (13) concerns research into the failure and repair of mining machinery and whether such repairs and overhauls return their performance to a condition that is "as good as new" in a reliability sense. To gain a better understanding of the effects which overhauling had on reliability characteristics of the mining machines, a detailed analysis of their times to failure for each overhaul rank was undertaken to discern those statistical distributions and their parameters which could accurately model their failure behavior. They were then compared to determine whether they exhibited increasing hazard rates, IHR, or constant hazard rates, CHR. The complete gearhead units under study exhibited Weibull shape parameters, β, with values as high as 2. De La Mare concluded that some component parts probably had β values possibly exceeding three or more. These high values of β indicated increasing hazard rates and therefore decreasing reliability with time. The quality of the data was insufficient to identify the critical components and their corresponding failure modes and causes.

Singhal and Amster (44) present a Markov process model which can be used to investigate cases where only a portion of circuit packs are nonrepairable (i.e. need to be replaced with new circuit packs). The model estimates the expected number of failures based on the infant mortality rate, age distribution of the installed circuit packs, growth potential, and fraction of nonrepairable circuit packs.

Work by Wong (52) describes findings leading to his conclusions on the "roller-coaster" shape characteristics for the hazard rate curve for electronics. Part one of his work (51) discussed problems with the bathtub curve as the shape of the failure rate curve. His work shows that electronic systems have generally decreasing failure rate curves with failure humps on them. The author calls these curves the "roller-coaster curves". The significance of the work is that it questions the traditional theory that assumes a constant hazard rate for electronic parts after an initial decreasing hazard rate or burn in period.

The work by Usher, et al (48), describes the development and implementation of a computerized reliability prediction model at an IBM facility. Through the analysis of historical life-test data, the model provides maximum likelihood estimates of the assumed Weibull life distributions of various types of electronic components. The resulting component life distribution estimates are used to predict the reliability of new electronic system configurations. The approach is based on the concept of estimating the reliability of components by observing their life in previously tested systems and then using these estimates to predict the reliability of a new configuration of these same components. The model allows for the analysis of a pooled set of life data, i.e. life data from different types of systems, to obtain component estimates.

Mann, et al (32), warn that the exponential distribution can be chosen as a failure distribution if and only if the assumption of a constant hazard rate can be justified. This assumption implies that the failures of a device is due, not to its deterioration as a result of wear, but to random shocks that occur according to the postulates of a Poisson Process.

O'Connor (40) explains that failures occurring in repairable systems are examples of a series of discrete events which occur randomly in a continuum. These situations, which are stochastic point processes, are analyzed using the statistics of <u>event series</u>.

The Poisson distribution function describes the situation in which events occur randomly and at a constant rate. This situation is described by a <u>homogeneous Pois-</u>

son process, HPP. An HPP is a stationary point process, since the distribution of the number of events in an interval of fixed length does not vary, regardless of when (where) the interval is sampled.

The Poisson distribution function is

$$f(x) = \frac{(\lambda x)^{\eta}}{\eta !} e^{-(\lambda x)} \qquad (\eta=0,1,2\ldots) \qquad (2.10)$$

where λ is the mean rate of occurrence, so that λx is the expected number of events in (0, x).

In a nonhomogeneous Poisson process (NHPP) the point process is non-stationary, so that the distribution of the number of events in an interval of fixed length changes as x increases. Typically, the discrete events (e.g. failures) might occur at an increasing or decreasing rate.

Note that an essential condition of any homogeneous Poisson process is that the probabilities of events occurring in any period are independent of what has occurred in preceding periods. An HPP describes a sequence of independently and identically exponentially distributed, IIED, random variables. A NHPP describes a sequence of random variables which is neither independently nor identically distributed. (40)

O'Connor gives techniques for determining whether a stochastic point process has a trend or not, that is, to determine whether a failure rate is increasing, decreasing, or constant. He describes the Centroid test or Laplace test. This technique involves analyzing arrival values of the event series. The arrival values, x_1 , x_2 ,....,x_η are the values of the independent variables (e.g. time) from x = 0 at which each event occurs. The interarrival values, X_1, X_2 ....., X_η are the intervals between successive events, 1, 2....., η, from x=0.

If x_0, is the period of observation, then the test statistic for trend is

$$U = \frac{\sum \frac{x_i}{\eta} - \frac{x_o}{2}}{x_o\sqrt{1/(12\eta)}} \tag{2.11}$$

This is called the Centroid test or the Laplace test. It compares the centroid of the observed arrival values with the mid-point of the period of observation. If U=0, there is no trend, i.e., the process is stationary. If U<0 the trend is decreasing, i.e., the interarrival values are tending to become larger. Conversely, when U>0 the trend is increasing, i.e., the interarrival values are tending to become progressively smaller.

O'Connor states that probability plotting methods to derive distribution parameters are only applicable when times to failure are independently and identically distributed, IID. This is usually the case for nonrepairable components and systems but may not be the case with failure data for repairable systems.

O'Connor explains that the continuous statistical distribution, which is likely to provide the best fit to a set of data, is not always readily apparent and he gives guidelines which should lead to the most revealing presentation.

Weibull parameters of the wearout failure mode can be derived if it is possible to identify the defective items and analyze their life and failure data separately from that of the non-defective items.

If we wish to understand the failure modes in order to make improvements, we must investigate all failures and analyze the distributions separately, since two or more failure modes may have markedly different parameters, yet overall failure data which are fitted by a distribution which is different to that of any of the underlying ones. (40)

For repairable systems, the distribution of times to first failure are much less important than is the failure rate or rate of occurrence of failures, ROCOF, of the system. Any repairable system may be considered as an assembly of parts, the parts being replaced when they fail. The methods of event series analysis can be used to analyze the system reliability.

Crow (7) explains that the homogeneous Poisson process is equivalent to the widely used Poisson distribution and exponential times-between-system failures model and is appropriate when the system's failure intensity is not affected by the system's age.

Crow's work is especially applicable to this author's research in that it is concerned with repairable systems. Most complex systems are repaired and not replaced when they fail. It is necessary to analyze the reliability characteristics of these type systems based on data generated under a customer use environment in order to assess mission reliability, frequency of failure, availability, or other parameters which may be influenced by the system's age. Crow's work (7) discusses the Weibull process or power law nonhomogeneous Poisson process model for analyzing the reliability of repairable systems.

Crow uses an extension of the homogeneous Poisson process known as the power law nonhomogeneous Poisson process which allows for the system failure intensity to change with system age.

In his work, Crow (7) assumed that the failures of each system under study are occurring according to a nonhomogeneous Poisson process with intensity function

$$u(t) = \lambda\beta t^{(\beta-1)} \tag{2.12}$$

where λ, $\beta > 0$ and t is the age of the system.

The nonhomogeneous Poisson process with intensity $u(t)$ has the power law mean value function

$$E[N(t)] = \lambda t^{\beta} \qquad t > 0 \tag{2.13}$$

which is the expected number of failures for a system during its age $(0, t)$.

To analyze the reliability of a complex repairable system based on failure data obtained from k copies of this system operated under the same environmental conditions, Crow (7) assumed that the failures for each of those k systems are governed by the intensity function given in equation (2.12).

$$u(t) = \lambda\beta t^{(\beta-1)}$$

The values of λ and β will be estimated based on data from the k systems.

Suppose the qth system is observed continuously from time S_q to T_q ($q=1\ldots k$). N_q is the number of failures experienced by the qth system and X_{iq} is the age of this system at the ith occurrence of failure. Then the maximum likelihood, ML, estimates of λ and β are values $\hat{\lambda}$ and $\hat{\beta}$ given by

$$\hat{\lambda} = \frac{\sum_{q=1}^{K} N_q}{\sum_{q=1}^{K}\left(T_q^{\hat{\beta}} - S_q^{\hat{\beta}}\right)}, \tag{2.14}$$

and

$$\hat{\beta} = \frac{\sum_{q=1}^{K} N_q}{\hat{\lambda} \sum_{q=1}^{K} \left(T_q^{\hat{\beta}} \ln T_q - S_q^{\hat{\beta}} \ln S_q \right) - \sum_{q=1}^{K} \sum_{i=1}^{N_q} \ln X_{iq}}. \tag{2.15}$$

In general, these equations cannot be solved explicitly for $\hat{\lambda}$ and $\hat{\beta}$ (the estimated values of λ and β), but must be solved by iterative procedures. Once the estimates $\hat{\lambda}$ and $\hat{\beta}$ are obtained the maximum likelihood estimate, MLE, of the intensity function can be calculated from the relationship

$$\hat{u}(t) = \hat{\lambda}\, \hat{\beta}\, t^{\hat{\beta}-1} \quad . \tag{2.16}$$

When $S_q=0$, and data are time truncated at $T_q=T$ ($q=1....k$) then the ML estimates $\hat{\lambda}$ and $\hat{\beta}$ are in closed form, that is,

$$\hat{\lambda} = \frac{\sum_{q=1}^{K} N_q}{K T^{\hat{\beta}}}, \tag{2.17}$$

and,

$$\hat{\beta} = \frac{\sum_{q=1}^{K} N_q}{\sum_{q=1}^{K} \sum_{i=1}^{N_q} \ln\left(\frac{T}{X_{iq}} \right)}. \tag{2.18}$$

Also, when $k=1$, $S_1=0$ and the data are failure truncated, that is , $X_{ni} = T_1$, then $\hat{\lambda}$ and $\hat{\beta}$ are in the closed form,

$$\hat{\lambda} = \frac{N_1}{T_1^{\beta}}, \tag{2.19}$$

and,

$$\hat{\beta} = \frac{N_1}{\sum_{i=1}^{N_i} \ln\left[\frac{T_1}{X_{i1}}\right]} . \tag{2.20}$$

Because of the functional forms of the intensity function and the mean value function, this particular nonhomogeneous Poisson process is often referred to as a "Weibull Poisson process", WPP, or the "power law Poisson process".

Note for $\beta=1$, one will have the homogeneous Poisson process . For $\beta>1$, u(t) is strictly increasing and the intervals between successive failures X_i- X_{i-1} are stochastically decreasing, which would be characteristic of a wearout situation. For $\beta<1$, u(t) is strictly decreasing and the intervals between successive failures X_i-X_{i-1} are stochastically increasing which would be characteristic of a debugging situation.

The multicomponent systems' failure data can be stochastically presented as a nonhomogeneous Poisson process, NHPP, with

$$V(t)= \lambda t^{\beta} \tag{2.21}$$

$$\text{where } V(t) = E[N(t)] . \tag{2.22}$$

The derivative $v(t)=V'(t)$ is the probability that a failure, not necessarily the first, occurs in (t, t+ dt). This contrasts sharply with the interpretation of the hazard function, h (x) dx, which is the conditional probability of first and only failure in (x,x+dx). This model states that the expected value of N(t) is λt^{β}.
In addition, it specifies the probability that N (t) will take on a specific value,

$$\Pr\{N(t) = j\} = \frac{(\lambda t^{\beta})^{j} e^{-\lambda t^{\beta}}}{j\,!} ,j = 0,1,2 \tag{2.23}$$

where N (t) is the number of failures which occur during (0, t).
{N (t), $t\geq 0$} is the integer valued counting process which includes both the number of failures in (0, t), N (t) and the instants T_1 ,T_2 .... at which they occur.

CHAPTER 3

OBJECTIVES AND RATIONALE

Failure systems can be categorized into two basic types: one time or nonrepairable systems and reusable or repairable systems. The term "system" here means a single component or a combination of components designed to perform a specific function. If continuous operation of the system is desired, then in the former case the system would be replaced by a new system upon failure. If failure data are available for a nonrepairable system, then, since the failure times are independent and identically distributed, the analyses involve the estimation of the corresponding life distribution. In the latter case, under continuous operation, the system is repaired, but not replaced, after each failure.

Any system which after failing to perform at least one of its required functions, can be restored to performing all of its required functions by any method, other than replacement of the entire system, is a <u>repairable</u> system.

For a repairable system, one is rarely interested primarily in time to first failure. Interest generally centers around the probability of system failure as a function of system age. The case studies included here concern repairable mechanical systems composed of non-repairable components. Crow's model (7) based on the nonhomogenous Poisson process (NHPP) was utilized to analyze failure data generated by multiple systems. The failure data for the systems can be combined since they are but samples of the same population.

O'Connor (40) explains that failures occurring in repairable systems are examples of a series of discrete events which occur randomly in a continuum. These situations which are stochastic point processes, are analyzed using the statistics of <u>event series</u>. The Crow (7) model or power law nonhomogeneous Poisson process is recognized by the reliability community as being the best model for repairable systems.

The Weibull distribution, on the other hand, is based on the hazard function or time to first failure of nonrepairable components. Since the failure times or hazard functions for components are independent and identically distributed, the Weibull distribution can be used to model the components' life distribution.

Simulation is the act of reproducing the behavior of a system. The events in a simulation can be generated in an unbiased way if random numbers are assigned to the events in the same proportions as their probability of occurrence. This process is known as Monte Carlo simulation because of the random numbers used to generate the simulation events.

Analytic models such as Crow's NHPP model are often quite restrictive in their assumptions. One advantage of a simulation, on the other hand, is the ability of the user to identify and specify relations of interest. In both cases one must find a selective representation of reality that captures the essence of the real problem.

According to Horowitz (23) a tractable mathematical model will generally be preferable to a simulation model. The purely mathematical models may require a higher level of abstraction than will comparable simulation models. When, therefore, the problem demands that realism not be sacrificed for a mathematical ideal whose distortions may in fact tend to mislead and misrepresent, simulation can provide the only operational means of analysis.

According to Hahn and Shapiro (18), "A major drawback of the Monte Carlo method is that there is frequently no way of determining whether any of the variables are dominant or more important than others." This points out the need for the important contribution that this work will make to reliability modelling. By combining the analytical abstraction of the Crow NHPP model with the practical realism of the Monte Carlo simulation using Weibull probability distributions of the component failure data the drawback to Monte Carlo simulation mentioned by Hahn and Shapiro (18) will be eliminated.

This technique of using Crow's NHPP model, which utilizes overall system (dirty) data to serve as a guide for testing the accuracy of a simulation using Weibull parameters of the individual component failure modes is new and unique. The simulations, based on fitting component failure data to a continuous distribution, are more valuable in that they give failure prediction results that can be attributed to individual components.

This work presents case studies on the analysis of reliability data for complex mechanical systems. The case studies are designed to test whether failure predictions made by the nonnomogenous Poisson process, the best known model for repairable systems, can be reasonably duplicated by a simulation that utilizes Weibull parameters of the major component failure modes. The individual component failures can be modeled by continuous statistical distributions such as the Weibull.

The Crow model determines when the overall system will be down, but the simulation predicts the frequency of failures of each component in a specific time frame. If the simulation identifies a finite number of parts that contribute to downtime, then, this information can be used to aid in the design of an optimum preventive maintenance program.

CHAPTER 4

METHOD AND PROCEDURE

A program was written to perform an iteration of Crow's equations for maximum likelihood (ML) estimates of λ and β using failure data from multiple systems that are representative of the same population. Crow's (7) techniques for combined failure data assume that each system is governed by the same nonhomogenous Poisson process failure intensity function:

$$u(t)=\lambda\beta t^{\beta-1}, \quad t>0 \tag{3.1}$$

where λ , $\beta > 0$ and t is the age of the system, in days.

The accuracy of this program was verified by using it to arrive at λ and β estimates using Crow's system data and examples from his papers. The systems in Crow's examples are failure truncated and, therefore, the ML estimates of λ and β are calculated from the following equations:

$$\hat{\lambda} = \frac{\sum_{q=1}^{k} N_q}{\sum_{q=1}^{k} \left(T_q^{\hat{\beta}} - S_q^{\hat{\beta}}\right)}, \text{ and} \tag{3.2}$$

$$\hat{\beta} = \frac{\sum_{q=1}^{k} N_q}{\hat{\lambda} \sum_{q=1}^{k} \left(T_q^{\hat{\beta}} \ln T_q - S_q^{\hat{\beta}} \ln S_q\right) - \sum_{q=1}^{k} \sum_{i=1}^{N_q} \ln X_{iq}}. \tag{3.3}$$

where: k = number of copies of the system,

q = the specific system copy, ($q = 1....k$),

S_q = starting time for system q,

N_q = number of failures experienced by the qth system,

X_{iq} = age of the system at the ith failure,in days,(i= 1...q), (q= 1...k), and

T_q = time of the last failure if failure is truncated, i.e., $X_{N_q,q} = T_q$, or the ending time of the system observation period if time truncated, i.e., $X_{N_q,q} < T_q$

The iteration program which is shown in the Appendix, was utilized to solve an example given in Crow's (7) paper. From the Crow data in the example, T_1=197.2, T_2=190.8, T_3=195.8, and the ML estimates of λ and β are $\hat{\lambda}$=0.443, and $\hat{\beta}$=0.626. These match Crow's results exactly and therefore verify the applicability of the iteration program.

Six different system failure databases were studied and are covered in the six following cases. Each of the six different system databases consists of data from at least two different, but essentially identical, versions of the particular system. Crow's (7) equations were developed to handle failure data generated by multiple systems that are representative of the same population. Therefore, the systems studied were chosen because of the existence of the multiple versions of such systems.

The first database analyzed was one for five different reactor systems of the same design, construction and application. This database was chosen because of the availability of failure data back to 1980. The Crow NHPP model requires that the failure data be available in uninterrupted segments. Database I was also chosen because of the availability of data on the individual component failures.

The second database studied was for failure data on five different, but identical, centrifugal compressors. Failure data on the systems (compressors) as a whole and their components were available since 1988.

The four other databases studied were all for failure data from reciprocating compressors that are different in either design, manufacturer or application. Data for these systems were also available from 1988.

It should be noted that 1988 was the year that the Maintenance Management Information System (MMIS) was implemented in the Louisiana Division of Dow Chemical Company. All of the data for all the databases except System I were obtained from the MMIS. The System I Database was derived from paper files and log books as well as the MMIS (1988 and after).

CASE 1. REPAIRABLE SYSTEMS DATABASE I

The first database considered in this research is called "Repairable Systems Database I" which consists of failure data on five reactor systems of the same design, construction and application. In Table 1 a total of nine systems and their corresponding failure dates are listed. The four added systems were brought about because the available failure data for four of the five reactor systems was not for a single continuous time period. The different time frames were therefore handled as different systems as per the methods described by Crow (7).

Failure data on System 1 was continuous for the period November 25, 1980 to November 26, 1986. From November 26, 1986 to September 22, 1987 the data was not used due to inconsistencies because of experimental work being conducted. Crow (7) states "The proper way to handle this situation is to treat the system data as if they were generated from two separate systems with failure data for each system over a continuous period of time." System 6, therefore, represents System 1 over the period from September 22, 1987 to February 22, 1991.

Similar procedures were followed for Systems 2 and 7, Systems 3 and 8, and Systems 5 and 9. System 4 did not have an interruption in the failure data accuracy.

As can be seen in Table 1, the date of each system failure is recorded along with the last date that the system was repaired. The days between failures are listed as well as the cumulative time that the system was under observation. Crow's NHPP model was applied to the failure data utilizing the iteration program to calculate the maximum likelihood estimates for β and λ for the nine systems involved. From the database, the times of the last failures are given in days as follows: $T_1=2301$, $T_2=3229$, $T_3=2840$, $T_4=3238$, $T_5=2753$, $T_6=3766$, $T_7=3719$, $T_8=3714$, $T_9=3767$. The maximum likelihood estimates of λ and β were determined to be $\hat{\lambda}=0.0001345$, $\hat{\beta}=1.411$.

TABLE 1: REPAIRABLE SYSTEMS DATABASE I

	EQUIP		LAST				EQUIP		LAST		
SYSTEM	ID	DATE	REPAIRED	BTWN	CUM	SYSTEM	ID	DATE	REPAIRED	BTWN	CUM
1	R-101	26-Nov-80	31-Oct-80	26	26	4	R-104	19-Oct-84	09-Mar-84	224	1449
1	R-101	08-Feb-82	26-Nov-80	440	466	4	R-104	21-Dec-85	19-Oct-84	428	1877
1	R-101	19-Jul-82	08-Feb-82	161	626	4	R-104	31-Aug-86	21-Dec-85	253	2130
1	R-101	10-Nov-83	19-Jul-82	479	1105	4	R-104	23-Feb-88	31-Aug-86	541	2671
1	R-101	30-Jul-84	10-Nov-83	263	1368	4	R-104	15-Mar-89	23-Feb-88	386	3057
1	R-101	21-Jun-85	30-Jul-84	326	1694	4	R-104	12-Sep-89	15-Mar-89	181	3238
1	R-101	04-Mar-86	21-Jun-85	256	1950						
1	R-101	28-Nov-86	04-Mar-86	267	2217	5	R-505	28-Feb-81	31-Oct-80	120	120
1	R-101	18-Feb-87	28-Nov-86	84	2301	5	R-505	17-Dec-81	28-Feb-81	292	412
						5	R-505	07-Nov-82	17-Dec-81	325	737
2	R-102	31-Oct-80	31-Oct-80	0	0	5	R-505	06-Nov-83	07-Nov-82	364	1101
2	R-102	04-Sep-81	31-Oct-80	308	308	5	R-505	17-May-84	06-Nov-83	193	1294
2	R-102	17-Dec-82	04-Sep-81	469	777	5	R-505	05-Dec-84	17-May-84	202	1496
2	R-102	01-Jan-84	17-Dec-82	380	1157	5	R-505	19-Dec-84	05-Dec-84	14	1510
2	R-102	26-Oct-84	01-Jan-84	299	1456	5	R-505	26-Feb-85	19-Dec-84	69	1579
2	R-102	22-Dec-85	26-Oct-84	422	1878	5	R-505	24-Oct-85	26-Feb-85	240	1819
2	R-102	23-Jan-87	22-Dec-85	397	2275	5	R-505	19-Dec-85	24-Oct-85	56	1875
2	R-102	16-Sep-87	23-Jan-87	236	2511	5	R-505	22-Dec-85	19-Dec-85	3	1878
2	R-102	04-Aug-88	16-Sep-87	323	2834	5	R-505	25-May-86	22-Dec-85	154	2032
2	R-102	03-Sep-89	04-Aug-88	395	3229	5	R-505	28-Feb-87	25-May-86	279	2311
						5	R-505	13-Dec-87	28-Feb-87	288	2599
3	R-103	13-Nov-80	31-Oct-80	13	13	5	R-505	15-May-88	13-Dec-87	154	2753
3	R-103	18-Dec-81	13-Nov-80	400	413						
3	R-103	02-Feb-83	18-Dec-81	411	824	6	R-101	22-Sep-87	31-Oct-80	2517	2517
3	R-103	05-Jun-84	02-Feb-83	489	1313	6	R-101	28-Sep-89	22-Sep-87	737	3254
3	R-103	27-Oct-84	05-Jun-84	144	1457	6	R-101	06-Jun-90	28-Sep-89	251	3505
3	R-103	07-Nov-84	27-Oct-84	11	1468	6	R-101	02-Dec-90	06-Jun-90	179	3684
3	R-103	04-Apr-85	07-Nov-84	148	1616	6	R-101	22-Feb-91	02-Dec-90	82	3766
3	R-103	30-Oct-86	04-Apr-85	574	2190						
3	R-103	10-Feb-87	30-Oct-86	103	2293	7	R-102	20-May-90	31-Oct-80	3488	3488
3	R-103	14-May-87	10-Feb-87	93	2386	7	R-102	06-Jan-91	20-May-90	231	3719
3	R-103	29-Jan-88	14-May-87	260	2646						
3	R-103	30-Jun-88	29-Jan-88	153	2799	8	R-103	20-May-90	31-Oct-80	3488	3488
3	R-103	10-Aug-88	30-Jun-88	41	2840	8	R-103	04-Dec-90	20-May-90	198	3686
						8	R-103	01-Jan-91	04-Dec-90	28	3714
4	R-104	10-Dec-80	31-Oct-80	40	40						
4	R-104	09-Apr-81	10-Dec-80	120	160	9	R-505	19-Aug-89	31-Oct-80	3214	3214
4	R-104	31-Jan-83	09-Apr-81	662	822	9	R-505	24-Dec-89	19-Aug-89	127	3341
4	R-104	07-Mar-83	31-Jan-83	35	857	9	R-505	27-May-90	06-Nov-89	202	3543
4	R-104	09-Mar-84	07-Mar-83	368	1225	9	R-505	06-Jan-91	27-May-90	224	3767

From these Crow parameters, the failure intensity function can be developed as well as the expected number of failures of the system. These are the predictions that will be used as a basis to verify whether the simulation approach can indeed predict the overall system failure rate.

The system failure intensity function is then estimated from the Crow equation number 3.1 by substituting the ML estimates of λ and β,

$$u(t) = \hat{\lambda}\,\hat{\beta}\,t^{\hat{\beta}-1} \tag{3.4}$$

substituting for $\hat{\lambda}$ and $\hat{\beta}$ gives

$$u(t) = (0.0001345)\ (1.411)\ t^{(1.411-1.0)}$$

$$u(t) = 0.00018978\ t^{(0.411)}$$

where $u(t)$ is the instantaneous failure intensity in failures per day.

Figure 1 is a plot of $u(t)$ over the period (0,1200). All times are in days. The increasing failure intensity is indicative of an increasing rate of occurrence of failures in the system.

A system modeled by an NHPP is deteriorating (improving) if the expected number of failures in any initial interval is no greater than (no less than) the expected number of failures in any interval of the same length occurring in a later interval. For $\beta > 1$, $u(t)$ is strictly increasing and the intervals between successive failures are stochastically decreasing , which is characteristic of a wearout situation.

The probability, $R\ (t\ )$, that a system of age t will successfully complete a mission of fixed duration $d > 0$ is called " mission reliability " and is defined by

$$R(t) = e^{-[\lambda(t+d)^{\beta} - \lambda(t)^{\beta}]} \tag{3.5}$$

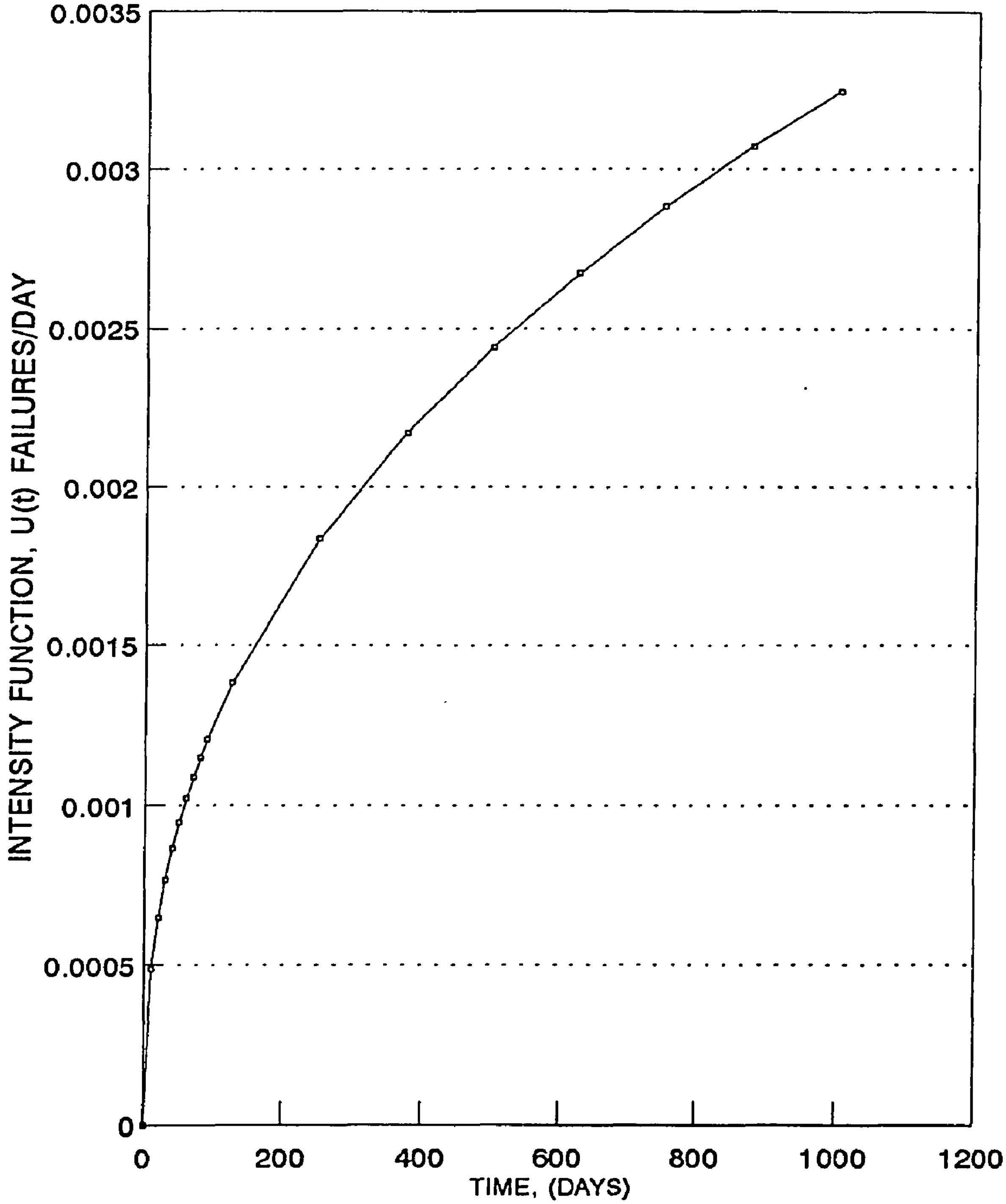


FIGURE 1: REPAIRABLE SYSTEM I FAILURE INTENSITY

In the plot shown in Figure 2, the mission duration, d, is assumed to be 365 days and the resulting equation is

$$R(t) = e^{-[0.0001345(t+365)^{1.411} - 0.001345(t)^{1.411}]} \tag{3.6}$$

For this plot, at time $t=0$, the system shows a 57.4 percent probability of not failing for 365 days or one year. The probability of the system remaining operative for another 365 days failure-free can be estimated, given the system is of age t, by equation number 3.6 utilizing the ML estimates for λ and β.

The expected number of failures of a system during its age (0, t) can be determined from the mean value function for the NHPP model by the relationship,

$$E[N(t)] = \lambda t^{\beta} \qquad t > 0 \tag{3.7}$$

This represents the integral of the area under the failure intensity curve (Figure 1) from time 0 to time t.

For Repairable System I, the expected number of failures in the first year was calculated to be 0.5547. Again, this data will be used to evaluate the simulation approach.

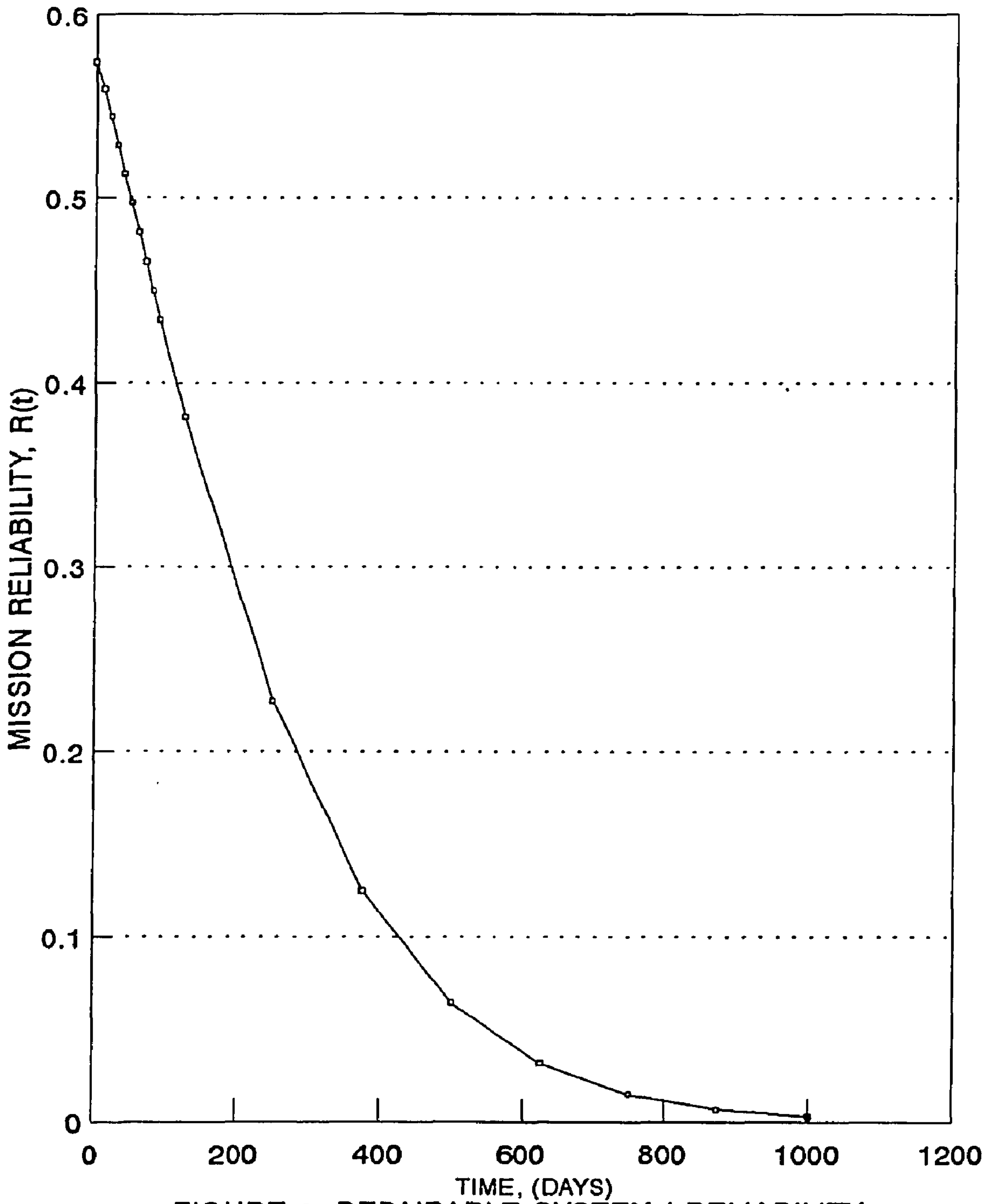


FIGURE 2: REPAIRABLE SYSTEM I RELIABILITY

Repairable System I represents reactor systems that are composed of four main components each of which is non-repairable and is replaced upon failure. Weibull analyses of the four major components of the reactor systems were done and the results are shown in Table 2. The Weibull charts are also shown for the four major components in Figures 3 through 6.

Component 2, center bearing, has the lowest characteristic life, η. This parameter, known as the scale parameter, corresponds to the life of 63.2 percent of the distribution of the failure data for that component and is derived from the relationship

$$F(t) = 1 - exp[-(t/\eta)^{\beta_W}] \tag{3.8}$$

when t is equal to η ,

$$F(t) = 1 - exp[-(1)]^{\beta_W}$$

$$= 1 - 0.368 = 0.632.$$

When t_O is added to η , the resulting life of Component 2 is predicted to be 15.98 months or 479.5 days.

All four of the major components show shape parameters, β_W , of greater than one and, therefore, all four components are in the wearout region of the failure curve.

The Weibull parameters from Table 2 were used as input in the Monte Carlo simulation program to determine the number of failures of the system predicted for a year and the number that would be contributed by each component failure mode. The number of failures predicted was 0.6 for the first year. The individual component contributions are included in Table 3. The overall prediction of 0.6 was the result of 99 simulation trials.

The prediction of 0.5547 failures for the first year by Crow's NHPP model agrees closely with the 0.6 failures predicted for the same time period by a Monte Carlo simulation of the four major components of the systems involved. The prediction of 1.475 failures for a period of two years for the total system by Crow's model also agrees closely with the Monte Carlo Simulation prediction of 1.6 failures for the same time period. The Monte Carlo Simulation Model predicts the number of failures to the system due to each component failure.

TABLE 2: WEIBULL PARAMETERS OF MAJOR COMPONENTS OF REPAIRABLE SYSTEM I

COMPONENT	SHAPE (β_W)	CHARACTERISTIC LIFE (η) (Months)	t_o
1	2.028	25.7	.099
2	3.198	15.99	2.864
3	1.859	44.83	.099
4	3.762	26.87	.099

TABLE 3: FAILURE PREDICTIONS BY MONTE CARLO SIMULATION FOR MAJOR COMPONENTS OF REPAIRABLE SYSTEM I

COMPONENT	NUMBER OF FAILURES PREDICTED FOR	
	One Year	Two Years
1	.1	.3
2	.3	1.0
3	.1	.1
4	.0	.1
Total System	.6	1.6

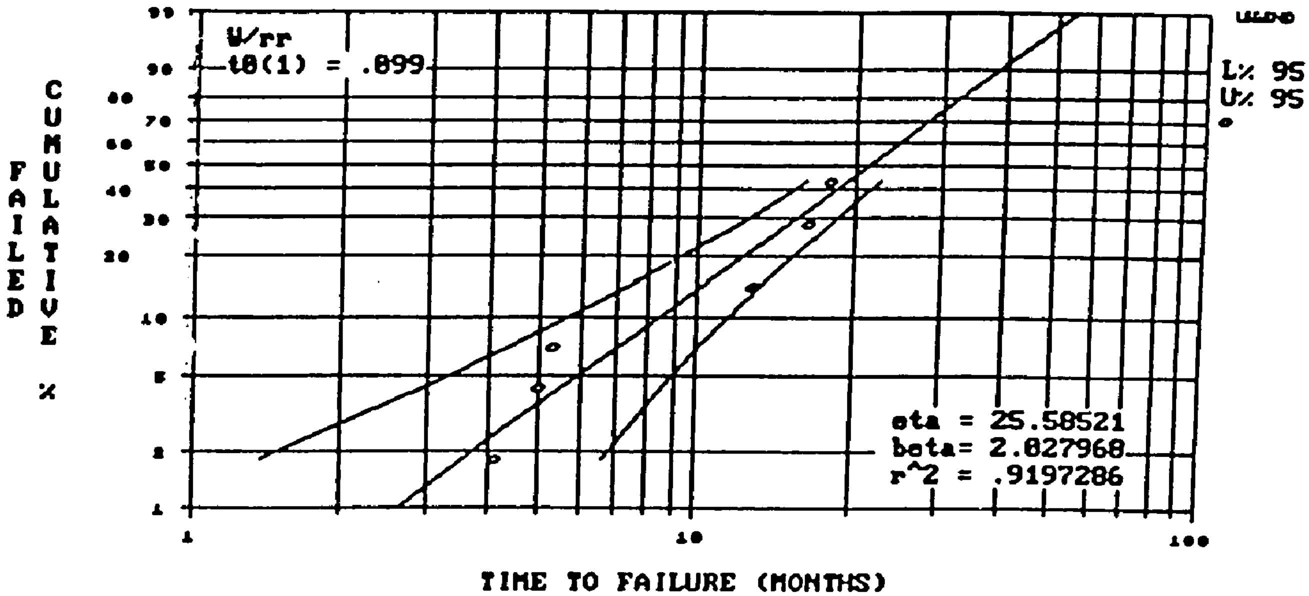


FIGURE 3: WEIBULL ANALYSIS FOR COMPONENT 1 SYSTEM I

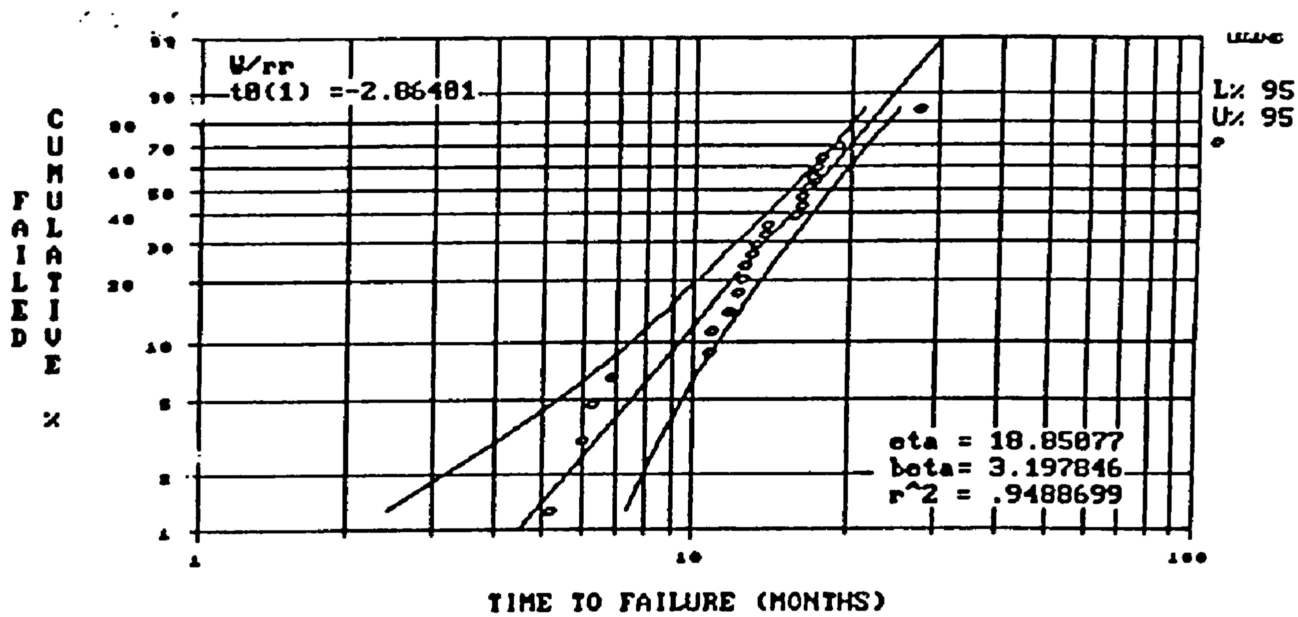


FIGURE 4: WEIBULL ANALYSIS FOR COMPONENT 2 SYSTEM I

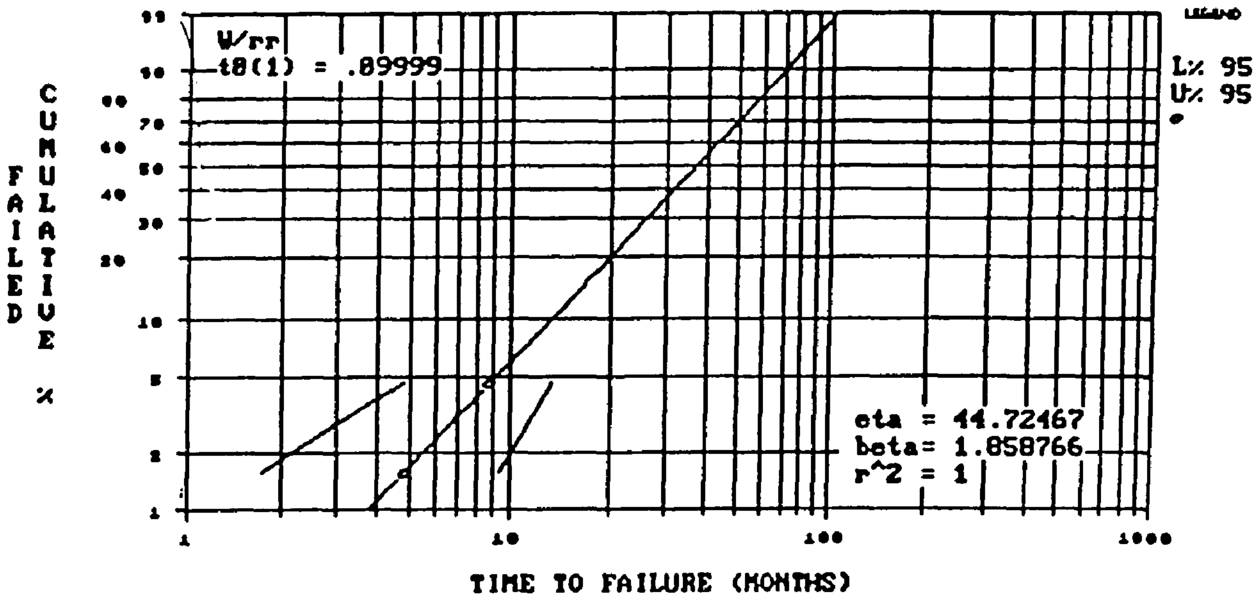


FIGURE 5: WEIBULL ANALYSIS OF COMPONENT 3 SYSTEM I

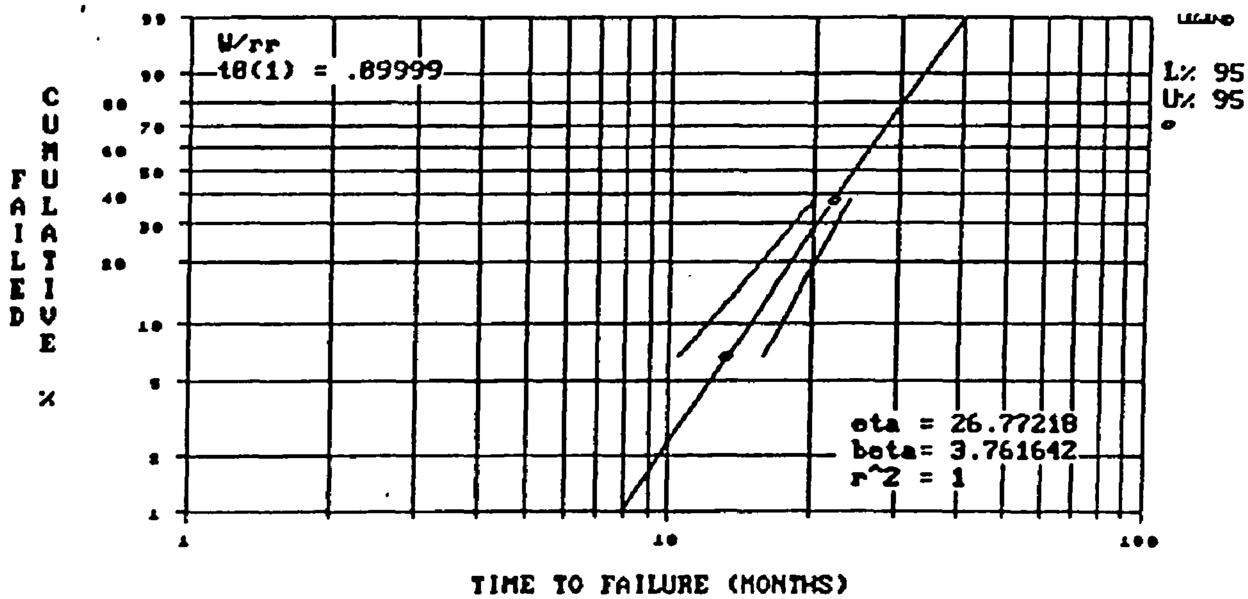


FIGURE 6: WEIBULL ANALYSIS OF COMPONENT 4 SYSTEM I

CASE 2. REPAIRABLE SYSTEM II DATABASE

The database for System II (Table 4) contains failure data for five different vertical compressor applications of the same compressor system. Time zero was started from April 26, 1988 and failure data is available through April 26, 1991.

The iteration program was utilized to obtain estimates of λ and β of $\hat{\lambda}=0.01093$ and $\hat{\beta}=0.912$.

Substituting for $\hat{\lambda}$ and $\hat{\beta}$ in the failure intensity function, equation number 3.1, yields

$$u\ (t) = (0.01093)\ (0.912)\ t^{(0.912-1.0)}$$

which reduces to

$$u\ (t) = (0.1093)\ (0.912)\ t^{(-0.088)}\ .$$

Figure 7 is a plot of $u\ (t)$ over the period (0 , 1000). This decreasing failure intensity is indicative of a decreasing rate of occurrence of failures to the system. For $\beta<1$, $u\ (t)$ is decreasing and the intervals between successive failures are stochastically increasing, which would be characteristic of an improving situation.

TABLE 4: REPAIRABLE SYSTEM II DATABASE

SYSTEM	EQUIP ID	DATE	LAST REPAIRED	BTWN	CUM.
1	C-1001A	26-Apr-88	26-Apr-88	0	0
1	C-1001A	13-May-88	26-Apr-88	17	17
1	C-1001A	22-Feb-89	13-May-88	285	302
1	C-1001A	17-Sep-89	22-Feb-89	207	509
1	C-1001A	16-Feb-90	17-Sep-89	152	661
1	C-1001A	22-Mar-90	16-Feb-90	34	695
2	C-1002A	13-May-88	26-Apr-88	17	17
2	C-1002A	1-Sep-88	13-May-88	111	128
2	C-1002A	29-Nov-88	1-Sep-88	89	217
2	C-1002A	20-Sep-89	29-Nov-88	295	512
2	C-1002A	28-Mar-90	20-Sep-89	189	701
2	C-1002A	31-Jul-90	28-Mar-90	125	826
2	C-1002A	7-Mar-91	31-Jul-90	219	1045
	C-1A	26-Sep-88	26-Apr-88	153	153
3	C-1A	12-Oct-88	26-Sep-88	16	169
3	C-1A	28-Nov-88	12-Oct-88	47	216
3	C-1A	19-Mar-89	28-Nov-88	111	327
3	C-1A	11-Aug-89	19-Mar-89	145	472
3	C-1A	1-Sep-89	11-Aug-89	21	493
3	C-1A	25-Jun-90	1-Sep-89	297	790
	C-2001A	23-Jun-88	26-Apr-88	58	58
4	C-2001A	22-Feb-89	23-Jun-88	244	302
4	C-2001A	1-Jun-89	22-Feb-89	99	401
4	C-2001A	2-Mar-90	1-Jun-89	274	675
5	C-2002A	2-Jul-88	26-Apr-88	67	67
5	C-2002A	5-Aug-88	2-Jul-88	34	101
5	C-2002A	5-Oct-88	5-Aug-88	61	162
5	C-2002A	13-Dec-90	5-Oct-88	799	961

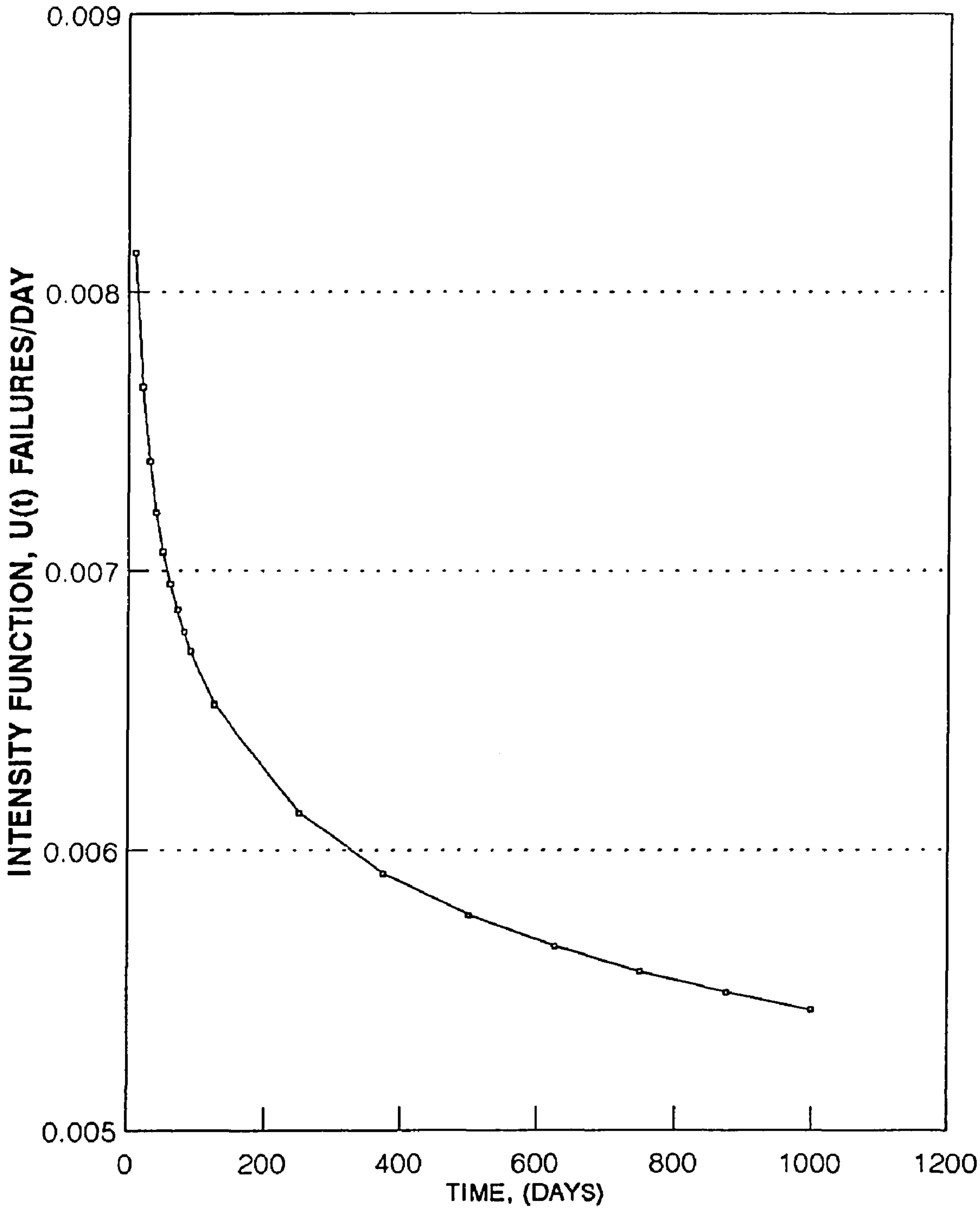


FIGURE 7: REPAIRABLE SYSTEM II FAILURE INTENSITY

The mission reliability is plotted in Figure 8 from the following equation :

$$R(t) = e^{-\left[0.01093(t+365)^{0.912} - 0.01093(t)^{0.912}\right]}$$

In this case the mission duration is 365 days. The probability that the system of age zero will successfully complete 365 days of operation without failure is only about 9.3 percent.

Mission reliability was plotted again in Figure 9 using a mission duration of 30 days. In this case, the probability, $R(t)$, that the system of age zero will successfully complete 30 days of operation without failure is 78.4 percent.

The expected number of failures of the system as a whole was calculated for one year using the relationship from equation number 3.7

$$E[N(t)] = \lambda t^{\beta} \qquad t > 0$$

and the result was 2.37 failures. The prediction for two years was 4.47 failures.

Repairable System II represents compressor systems that are made up of three main components each of which is non-repairable and is replaced upon failure. Weibull analyses of the three components of System II were made and the resulting parameters are shown in Table 5. The Weibull charts are also shown for the three components in Figures 10 through 12. These three components were chosen because the failure data from the work order system indicated that they were the major contributors to failures of the system. The three components chosen were bearings, gears and seals.

As before, the Weibúll parameters were used as input in the Monte Carlo simulation program to determine the number of incidents predicted for one year and two years, respectively. The number of failures predicted for one year was 2.6 and for two years was 6.0. Component failure predictions are included in Table 6.

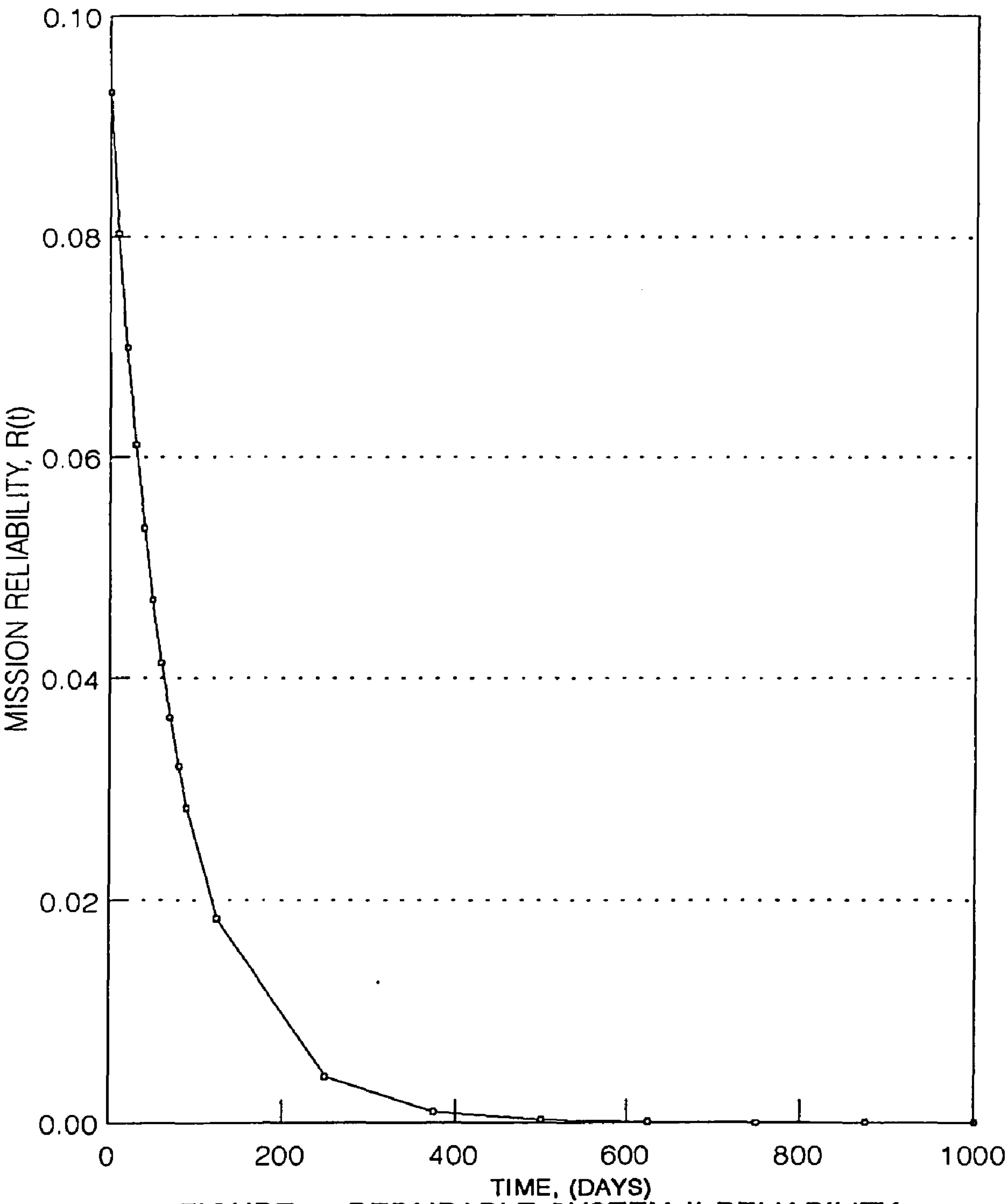


FIGURE 8: REPAIRABLE SYSTEM II RELIABILITY

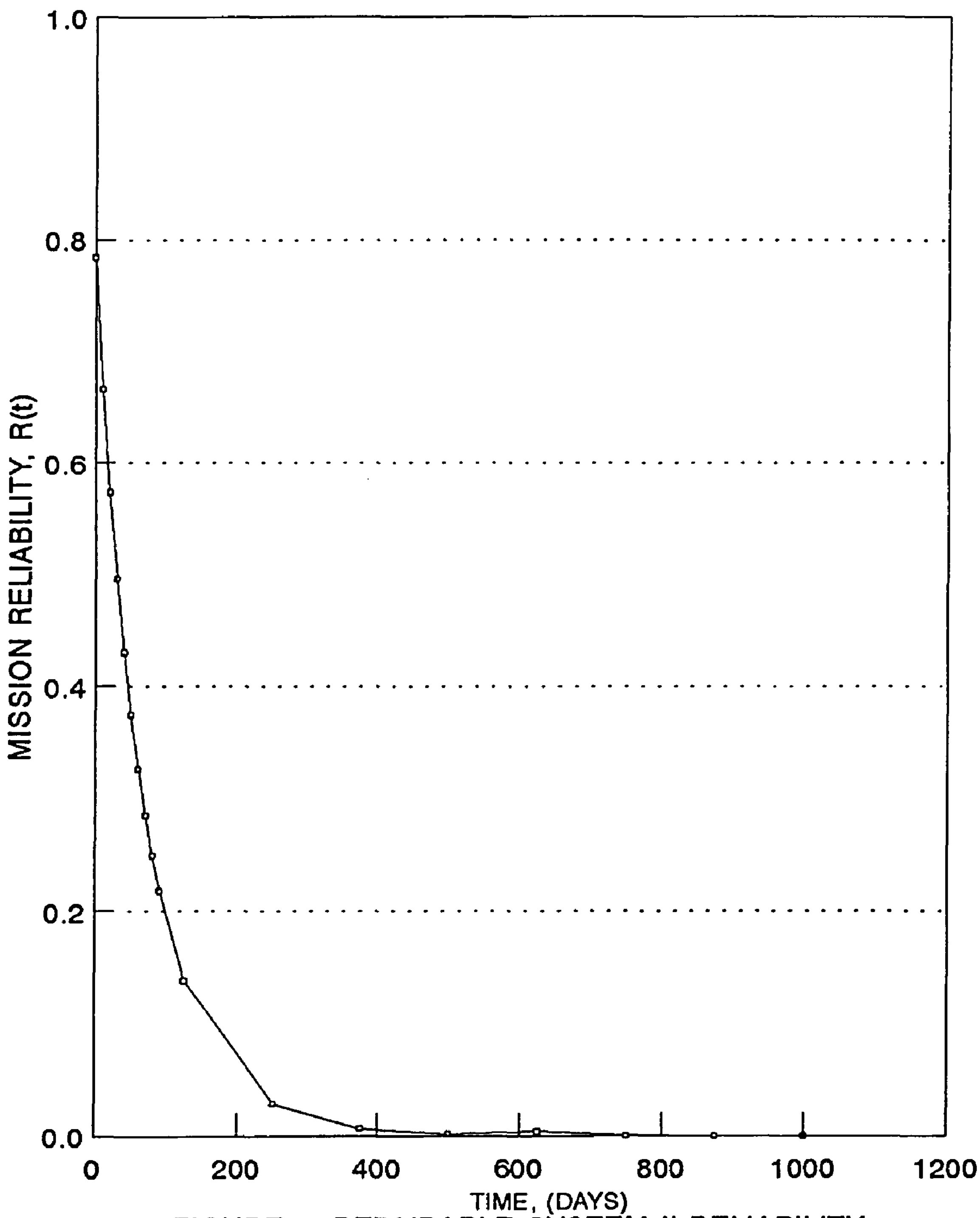


FIGURE 9: REPAIRABLE SYSTEM II RELIABILITY

TABLE 5: WEIBULL PARAMETERS OF MAJOR COMPONENTS OF REPAIRABLE SYSTEM II

COMPONENT	SHAPE (β_W)	CHARACTERISTIC LIFE (η) (Days)	t_0
1	1.1833	264.43	-0.302
2	1.9683	328.61	15.99
3	2.0448	198.14	15.99

TABLE 6: FAILURE PREDICTIONS BY MONTE CARLO SIMULATION FOR MAJOR COMPONENTS OF REPAIRABLE SYSTEM II

COMPONENT	NUMBER OF FAILURES PREDICTED FOR	
	One Year	Two Years
1	1.1	2.5
2	.4	1.1
3	1.1	2.3
Total System	2.6	6.0

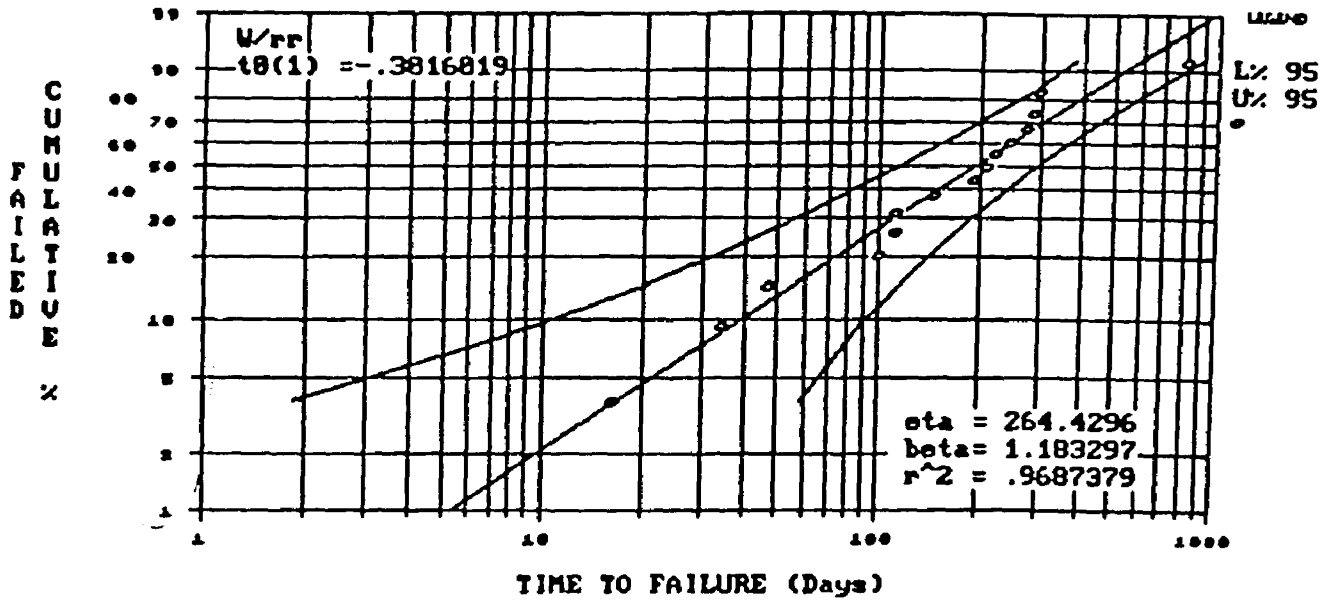


FIGURE 10: WEIBULL ANALYSIS FOR COMPONENT 1 SYSTEM II

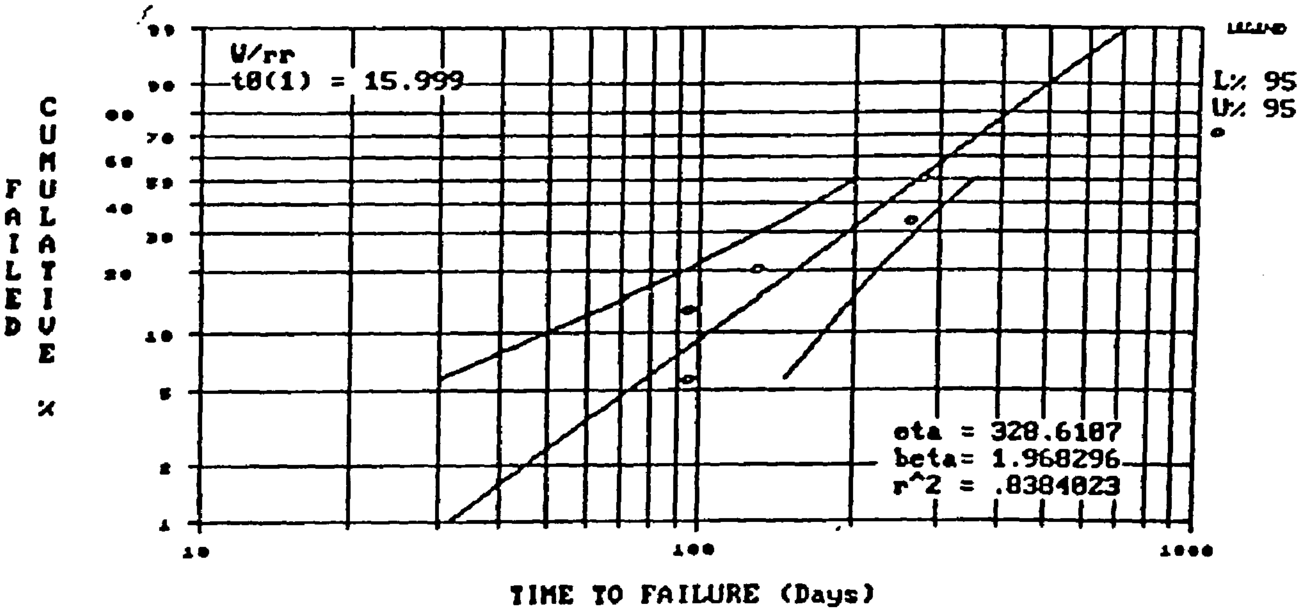


FIGURE 11: WEIBULL ANALYSIS FOR COMPONENT 2 SYSTEM II

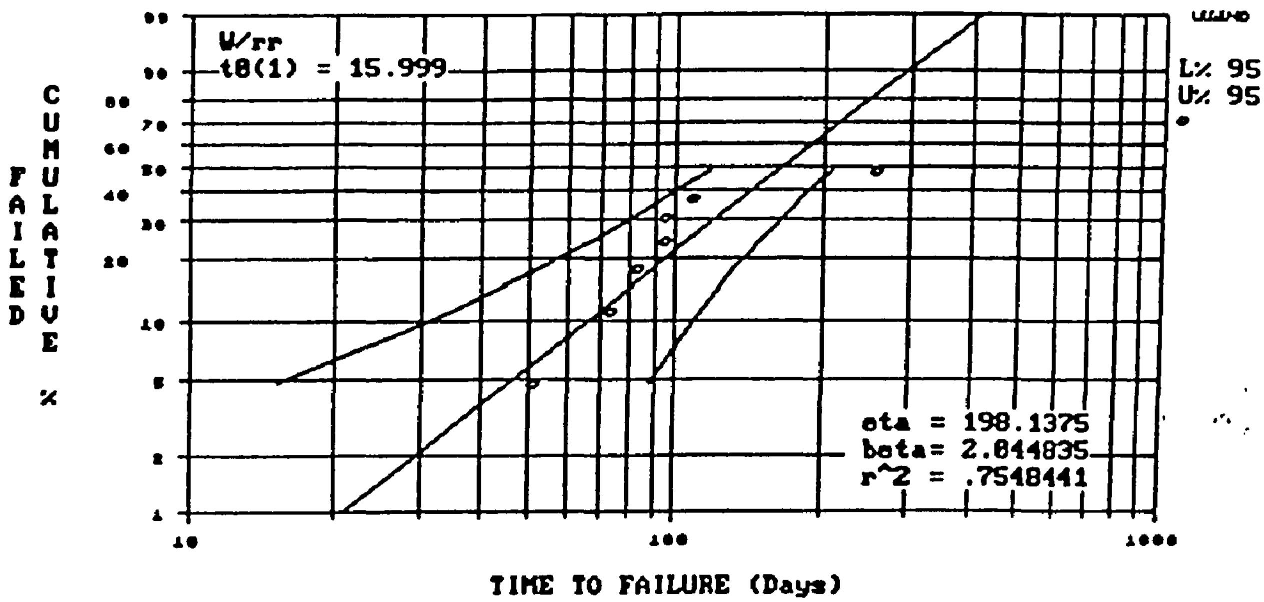


FIGURE 12: WEIBULL ANALYSIS FOR COMPONENT 3 SYSTEM II

CASE 3. REPAIRABLE SYSTEM III DATABASE

The database for Repairable System III (Table 7) contains data for five single stage reciprocating compressors. Time zero initialized on December 15, 1987 and failure data is available through February 12, 1991.

As previously described, the iteration program was used to arrive at the Crow NHPP model and the estimates of λ and β of $\hat{\lambda}$ =0.00365292 and $\hat{\beta}$=1.218006. Plots of failure intensity, $u(t)$, and mission reliability, $R(t)$, are shown in Figures 13 through 15.

The power law mean value function or the expected number of failures of System III was calculated to be 4.83 for the first year and 11.22 for two years.

Repairable System III represents compressor systems that are composed of many components, four of which were chosen as contributing to the majority of the failures. Weibull analyses of failure data for these four components were done and the parameters are given in Table 8. The Weibull charts are also shown for these four components in Figures 16 through 19.

The Weibull parameters were used in the Monte Carlo Simulation to predict the number of failures for one year and two years, respectively. The number of failures for one year was predicted to be 5.1. The prediction for two years was 10.4. The individual component contributions are included in Table 9.

TABLE 7: REPAIRABLE SYSTEMS DATABASE III

	EQUIP		LAST				EQUIP		LAST		
SYSTEM	ID	DATE	REPAIRED	BTWN	CUM.	SYSTEM	ID	DATE	REPAIRED	BTWN	CUM.
1	C-101	18-Dec-87	16-Dec-87	1	1	3	C-103	15-Feb-90	05-Feb-90	10	794
1	C-101	12-Jan-88	16-Dec-87	27	28	3	C-103	20-Feb-90	15-Feb-90	4	798
1	C-101	13-Jan-88	12-Jan-88	1	29	3	C-103	09-Apr-90	20-Feb-90	48	846
1	C-101	18-Jan-88	13-Jan-88	5	34	3	C-103	30-Apr-90	09-Apr-90	21	867
1	C-101	05-Feb-88	18-Jan-88	18	52	3	C-103	09-May-90	30-Apr-90	9	876
1	C-101	04-May-88	05-Feb-88	89	141	3	C-103	18-Jun-90	09-May-90	40	916
1	C-101	18-May-88	04-May-88	14	155	3	C-103	19-Jun-90	18-Jun-90	1	917
1	C-101	09-Sep-88	18-May-88	114	269	3	C-103	26-Jun-90	19-Jun-90	7	924
1	C-101	29-Sep-88	09-Sep-88	20	289	3	C-103	29-Aug-90	26-Jun-90	64	988
1	C-101	02-Dec-88	29-Sep-88	64	353	3	C-103	28-Sep-90	29-Aug-90	30	1018
1	C-101	23-Mar-89	02-Dec-88	111	464	3	C-103	14-Dec-90	28-Sep-90	77	1095
1	C-101	20-Jun-89	23-Mar-89	89	553	3	C-103	11-Feb-91	14-Dec-90	59	1154
1	C-101	07-Aug-89	20-Jun-89	48	601						
1	C-101	08-Aug-89	07-Aug-89	1	602	4	C-104	02-Apr-88	15-Dec-87	109	109
1	C-101	21-Sep-89	08-Aug-89	44	646	4	C-104	26-Aug-88	02-Apr-88	146	255
1	C-101	27-Jul-90	21-Sep-89	309	955	4	C-104	18-Nov-88	26-Aug-88	84	339
1	C-101	01-Aug-90	27-Jul-90	5	960	4	C-104	16-Jan-89	18-Nov-88	59	398
1	C-101	09-Aug-90	01-Aug-90	8	968	4	C-104	17-Jan-89	16-Jan-89	1	399
1	C-101	16-Aug-90	09-Aug-90	7	975	4	C-104	18-Feb-89	17-Jan-89	32	431
1	C-101	23-Aug-90	16-Aug-90	7	982	4	C-104	09-Mar-89	18-Feb-89	19	450
1	C-101	08-Nov-90	23-Aug-90	76	1057	4	C-104	15-Feb-90	09-Mar-89	343	793
1	C-101	08-Nov-90	08-Nov-90	2	1059	4	C-104	16-Feb-90	15-Feb-90	1	794
1	C-101	15-Nov-90	08-Nov-90	7	1066	4	C-104	21-Mar-90	16-Feb-90	33	827
1	C-101	20-Nov-90	15-Nov-90	5	1071	4	C-104	09-Apr-90	21-Mar-90	19	846
1	C-101	29-Jan-91	20-Nov-90	70	1141	4	C-104	15-Jun-90	09-Apr-90	67	913
1	C-101	12-Feb-91	29-Jan-91	14	1155	4	C-104	29-Aug-90	15-Jun-90	75	988
						4	C-104	08-Nov-90	29-Aug-90	72	1060
2	C-102	15-Dec-87	15-Dec-87	0	0	4	C-104	18-Nov-90	08-Nov-90	10	1070
2	C-102	14-Jan-88	15-Dec-87	30	30	4	C-104	14-Dec-90	18-Nov-90	26	1096
2	C-102	02-Mar-88	14-Jan-88	48	78						
2	C-102	28-May-88	02-Mar-88	88	166	5	C-105	15-Dec-87	15-Dec-87	0	0
2	C-102	05-Oct-88	28-May-88	130	296	5	C-105	08-Jan-88	15-Dec-87	24	24
2	C-102	10-Jan-89	05-Oct-88	96	392	5	C-105	13-Jan-88	08-Jan-88	5	29
2	C-102	22-Jun-89	10-Jan-89	163	555	5	C-105	10-Jan-89	13-Jan-88	363	392
2	C-102	02-Jan-90	22-Jun-89	194	749	5	C-105	24-Jan-89	10-Jan-89	14	406
2	C-102	18-Jul-90	02-Jan-90	197	946	5	C-105	18-Feb-89	24-Jan-89	25	431
2	C-102	09-Aug-90	18-Jul-90	22	968	5	C-105	28-Mar-89	18-Feb-89	38	469
2	C-102	08-Nov-90	09-Aug-90	91	1059	5	C-105	18-Apr-89	28-Mar-89	21	490
2	C-102	14-Dec-90	08-Nov-90	36	1095	5	C-105	31-Aug-89	18-Apr-89	135	625
2	C-102	21-Jan-91	14-Dec-90	38	1133	5	C-105	29-Sep-89	31-Aug-89	29	654
2	C-102	28-Jan-91	21-Jan-91	7	1140	5	C-105	06-Oct-89	29-Sep-89	7	661
2	C-102	11-Feb-91	28-Jan-91	14	1154	5	C-105	11-Oct-89	06-Oct-89	5	666
						5	C-105	13-Oct-89	11-Oct-89	2	668
3	C-103	17-Dec-87	15-Dec-87	2	2	5	C-105	31-Oct-89	13-Oct-89	18	686
3	C-103	10-Jan-88	17-Dec-87	24	26	5	C-105	02-Feb-90	31-Oct-89	94	780
3	C-103	11-Nov-88	10-Jan-88	306	332	5	C-105	09-Feb-90	02-Feb-90	7	787
3	C-103	14-Feb-89	11-Nov-88	95	427	5	C-105	10-Jul-90	09-Feb-90	151	938
3	C-103	25-Apr-89	14-Feb-89	71	498	5	C-105	01-Aug-90	10-Jul-90	22	960
3	C-103	08-Sep-89	25-Apr-89	136	634	5	C-105	09-Aug-90	01-Aug-90	8	968
3	C-103	24-Jan-90	08-Sep-89	137	771	5	C-105	23-Aug-90	09-Aug-90	14	982
3	C-103	01-Feb-90	24-Jan-90	8	779	5	C-105	19-Nov-90	23-Aug-90	88	1070
3	C-103	05-Feb-90	01-Feb-90	5	784						

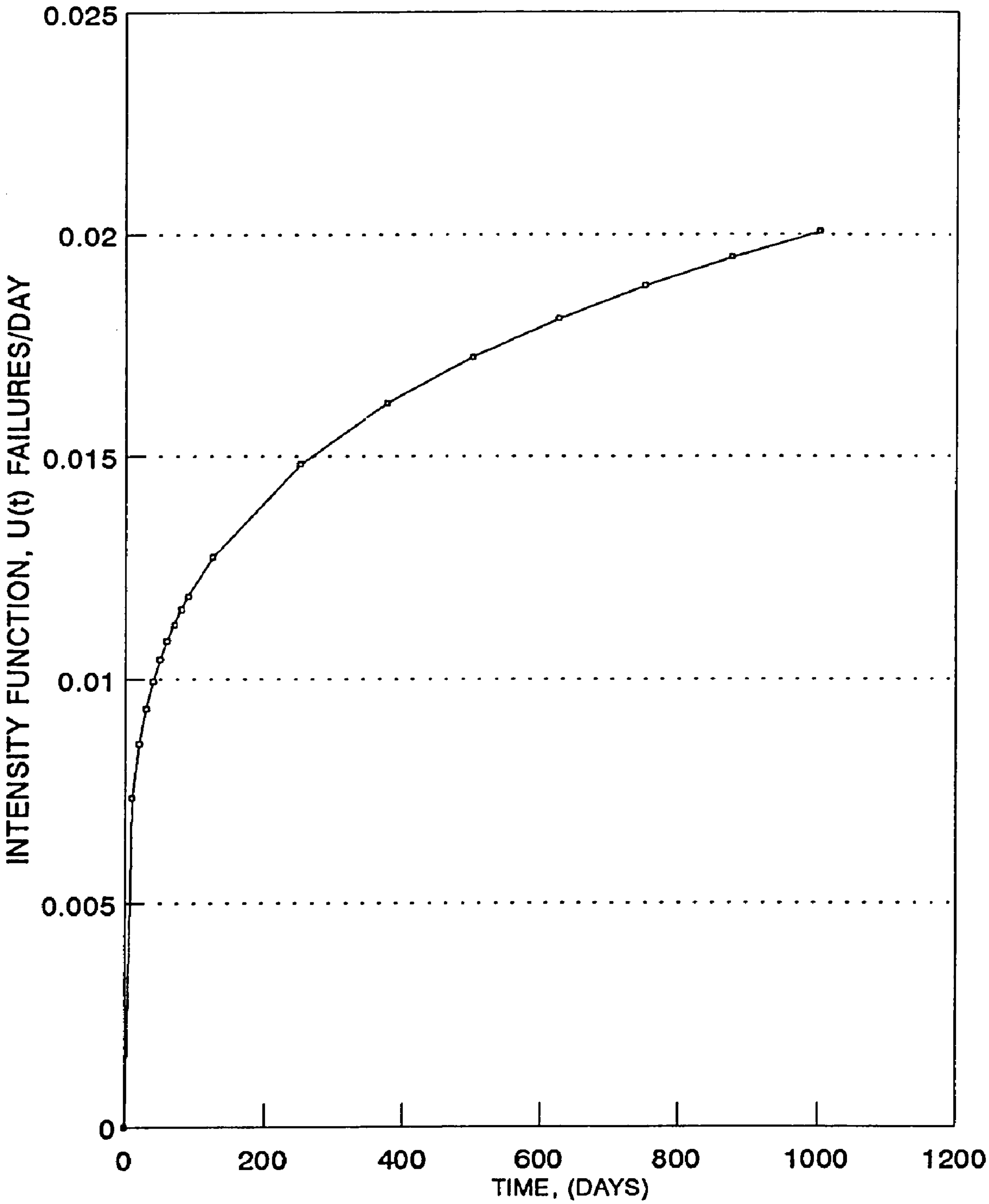


FIGURE 13: REPAIRABLE SYSTEM III FAILURE INTENSITY

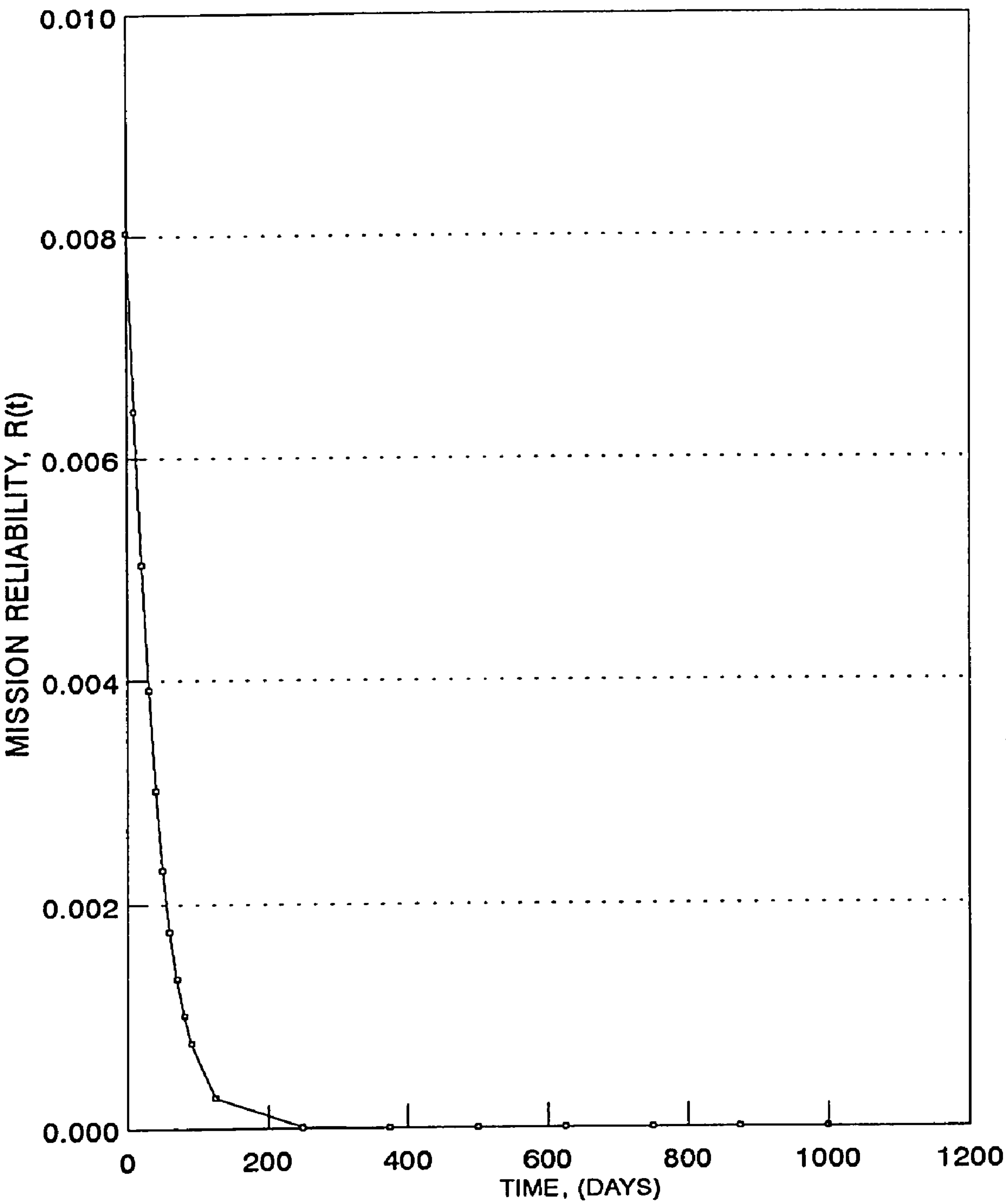


FIGURE 14: REPAIRABLE SYSTEM III RELIABILITY

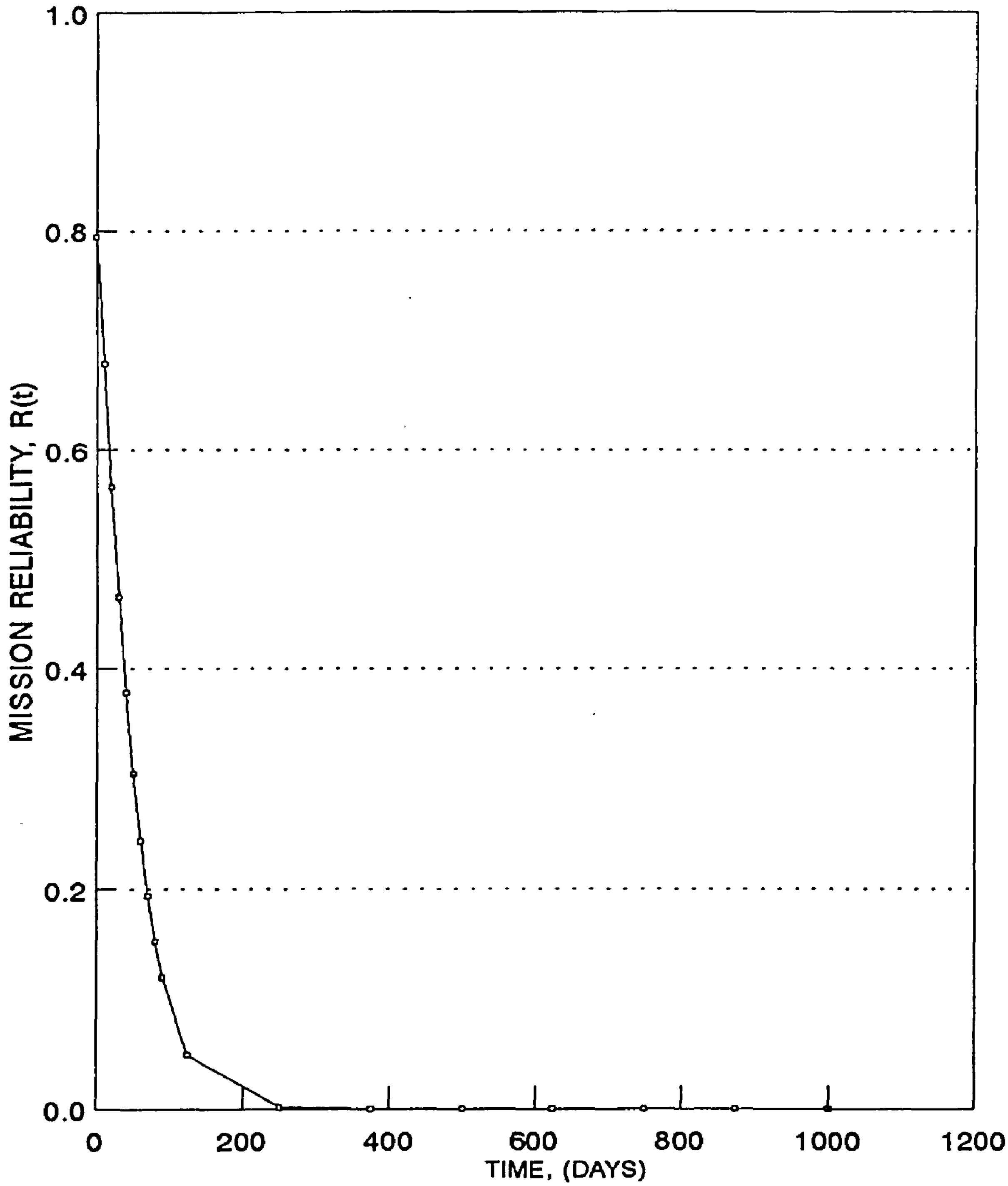


FIGURE 15: REPAIRABLE SYSTEM III RELIABILITY

TABLE 8: WEIBULL PARAMETERS OF MAJOR COMPONENTS OF REPAIRABLE SYSTEM III

COMPONENT	SHAPE (β_W)	CHARACTERISTIC LIFE (η) (Days)	t_0
1	1.2649	208.72	.999
2	0.4945	208.03	391.21
3	0.8334	249.86	-2.229
4	1.6126	881.38	-178.802

TABLE 9: FAILURE PREDICTIONS BY MONTE CARLO SIMULATION FOR MAJOR COMPONENTS OF REPAIRABLE SYSTEM III

COMPONENT	NUMBER OF FAILURES PREDICTED FOR	
	One Year	Two Years
1	1.5	3.0
2	1.6	3.3
3	1.8	3.7
4	.2	.4
Total System	5.1	10.4

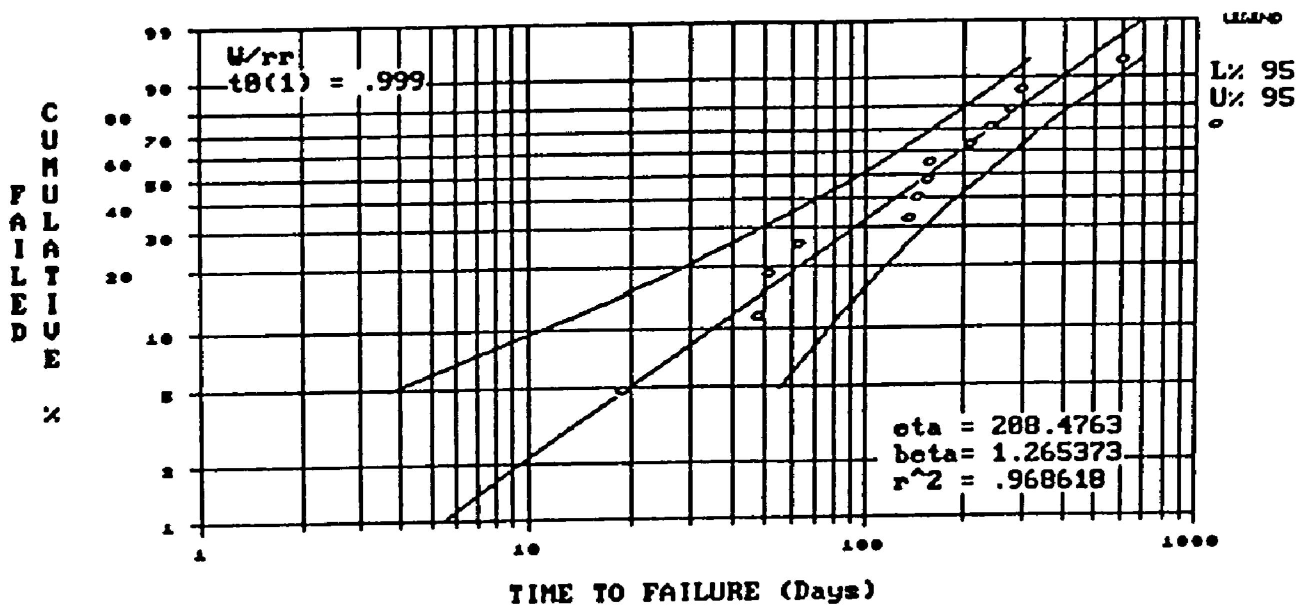


FIGURE 16: WEIBULL ANALYSIS FOR COMPONENT 1 SYSTEM III

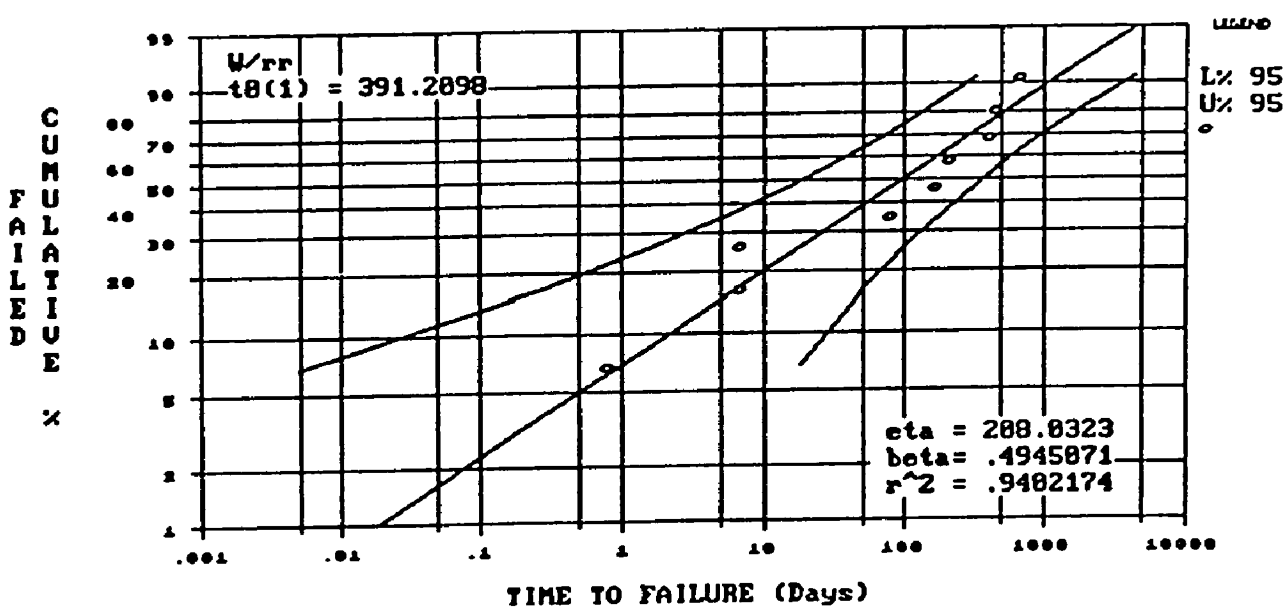


FIGURE 17: WEIBULL ANALYSIS FOR COMPONENT 2 SYSTEM III

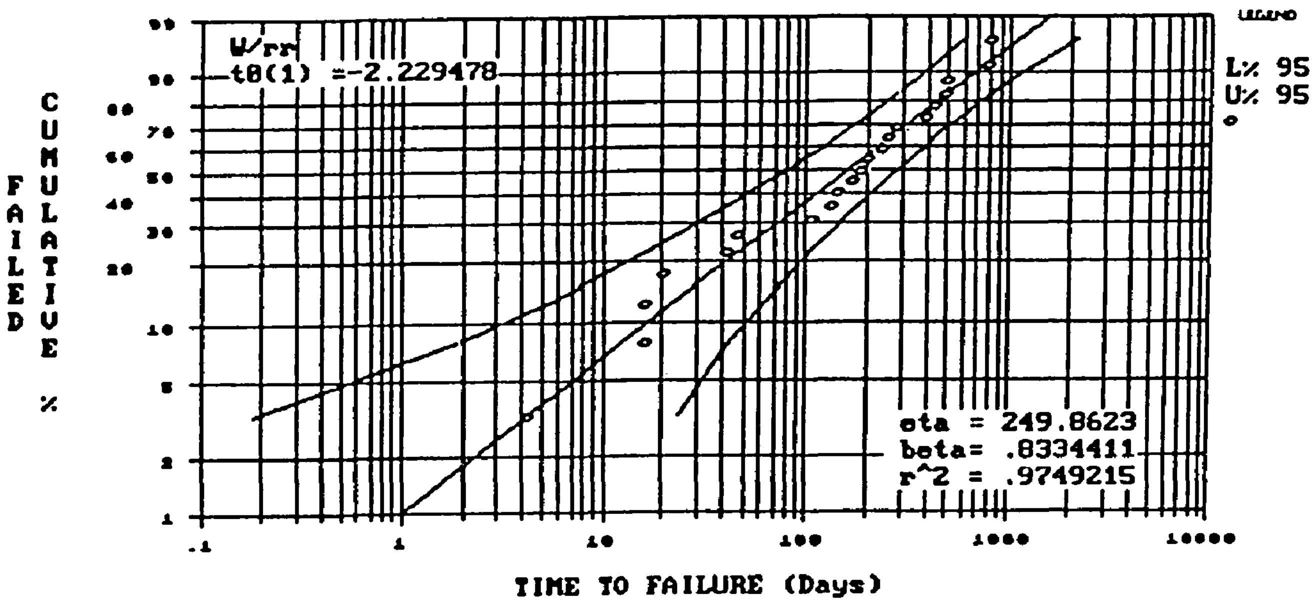


FIGURE 18: WEIBULL ANALYSIS FOR COMPONENT 3 SYSTEM III

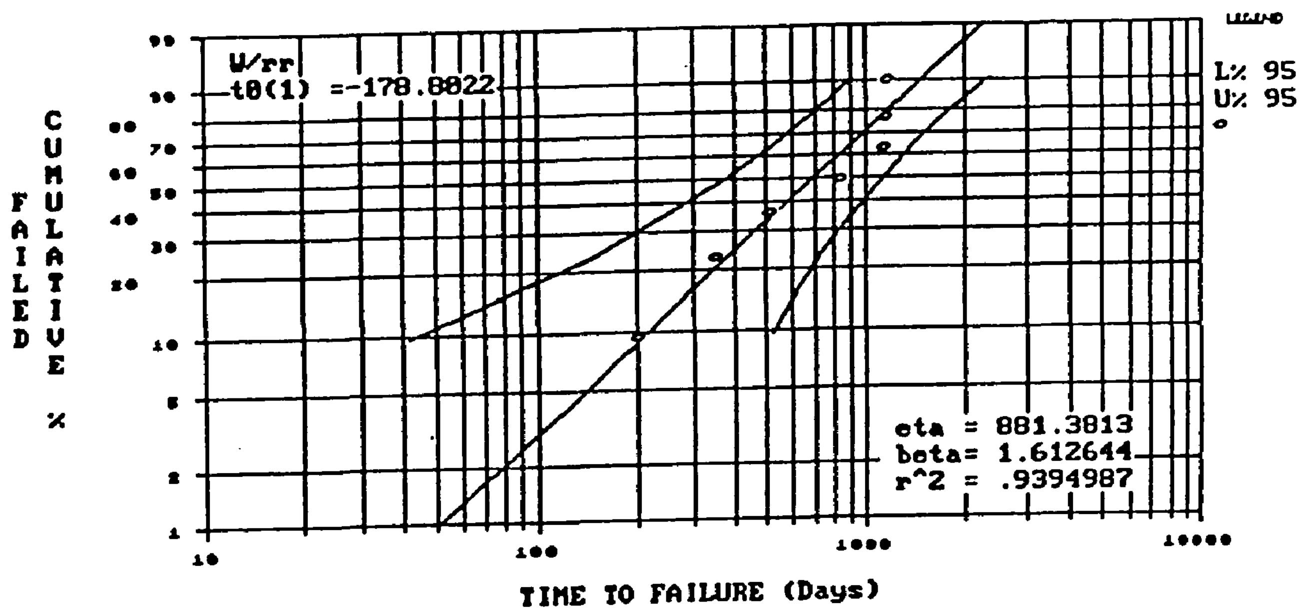


FIGURE 19: WEIBULL ANALYSIS FOR COMPONENT 4 SYSTEM III

CASE 4. REPAIRABLE SYSTEM IV DATABASE

The database for Repairable System IV (Table 10) consists of two reciprocating compressors that are two staged. Time zero initiated on January 5, 1988 and failure data is available through November 28, 1990.

The iteration program was utilized to arrive at the Crow NHPP estimates of λ and β of $\hat{\lambda}=.004706935$ and $\hat{\beta}=1.30201$.

The Crow NHPP predictions were calculated to be 10.206 failures for one year and 25.166 failures for two years.

Repairable System IV major component failure modes were fitted to Weibull distributions and the parameters are given in Table 11. The Weibull charts for these four major components are shown in Figures 20 through 23.

The Monte Carlo predictions using the Weibull parameters were determined to be 8.8 failures for one year and 17.5 failures for two years. The individual component contributions are given in Table 12.

TABLE 10: REPAIRABLE SYSTEMS DATABASE IV

SYSTEM	EQUIP ID	DATE	LAST REPAIRED	BTWN	CUM	SYSTEM	EQUIP ID	DATE	LAST REPAIRED	BTWN	CUM
1	C-108	05-Jan-88	05-Jan-88	0	0	1	C-108	07-May-90	30-Apr-90	7	853
1	C-108	26-Jan-88	05-Jan-88	21	21	1	C-108	29-Aug-90	07-May-90	114	967
1	C-108	12-Feb-88	26-Jan-88	17	38	1	C-108	18-Sep-90	29-Aug-90	20	987
1	C-108	18-Mar-88	12-Feb-88	35	73	1	C-108	10/22/90	18-Sep-90	34	1021
1	C-108	21-Mar-88	18-Mar-88	3	76	1	C-108	11/21/90	10/22/90	30	1051
1	C-108	19-Apr-88	21-Mar-88	29	105	1	C-108	11/28/90	11/21/90	7	1058
1	C-108	05-Aug-88	19-Apr-88	108	213						
1	C-108	01-Sep-88	05-Aug-88	27	240	2	C-109	01/19/88	01/05/88	14	14
1	C-108	08-Sep-88	01-Sep-88	7	247	2	C-109	07/07/88	01/19/88	170	184
1	C-108	28-Sep-88	08-Sep-88	20	267	2	C-109	07/10/88	07/07/88	3	187
1	C-108	23-Nov-88	28-Sep-88	56	323	2	C-109	08/08/88	07/10/88	29	216
1	C-108	08-Dec-88	23-Nov-88	15	338	2	C-109	09/12/88	08/08/88	35	251
1	C-108	12-Dec-88	08-Dec-88	4	342	2	C-109	09/21/88	09/12/88	9	260
1	C-108	19-Dec-88	12-Dec-88	7	349	2	C-109	09/30/88	09/21/88	9	269
1	C-108	27-Dec-88	19-Dec-88	8	357	2	C-109	10/27/88	09/30/88	27	296
1	C-108	30-Dec-88	27-Dec-88	3	360	2	C-109	12/06/88	10/27/88	40	336
1	C-108	13-Mar-89	30-Dec-88	73	433	2	C-109	01/26/89	12/06/88	51	387
1	C-108	27-Mar-89	13-Mar-89	14	447	2	C-109	03/27/89	01/26/89	60	447
1	C-108	26-Apr-89	27-Mar-89	30	477	2	C-109	06/22/89	03/27/89	87	534
1	C-108	02-May-89	26-Apr-89	6	483	2	C-109	08/31/89	06/22/89	70	604
1	C-108	22-May-89	02-May-89	20	503	2	C-109	01/02/90	08/31/89	124	728
1	C-108	25-May-89	22-May-89	3	506	2	C-109	01/25/90	01/02/90	23	751
1	C-108	26-May-89	25-May-89	1	507	2	C-109	02/02/90	01/25/90	8	759
1	C-108	01-Jun-89	26-May-89	6	513	2	C-109	02/06/90	02/02/90	4	763
1	C-108	02-Jun-89	01-Jun-89	1	514	2	C-109	02/20/90	02/06/90	14	777
1	C-108	02-Aug-89	02-Jun-89	61	575	2	C-109	02/26/90	02/20/90	6	783
1	C-108	09-Aug-89	02-Aug-89	7	582	2	C-109	02/27/90	02/26/90	1	784
1	C-108	12-Aug-89	09-Aug-89	3	585	2	C-109	03/06/90	02/27/90	7	791
1	C-108	12-Sep-89	12-Aug-89	31	616	2	C-109	03/07/90	03/06/90	1	792
1	C-108	13-Sep-89	12-Sep-89	1	617	2	C-109	03/12/90	03/07/90	5	797
1	C-108	19-Sep-89	13-Sep-89	6	623	2	C-109	04/09/90	03/12/90	28	825
1	C-108	21-Sep-89	19-Sep-89	2	625	2	C-109	04/25/90	04/09/90	16	841
1	C-108	18-Oct-89	21-Sep-89	27	652	2	C-109	04/27/90	04/25/90	2	843
1	C-108	19-Oct-89	18-Oct-89	1	653	2	C-109	06/29/90	04/27/90	63	906
1	C-108	24-Oct-89	19-Oct-89	5	658	2	C-109	07/17/90	06/29/90	18	924
1	C-108	27-Oct-89	24-Oct-89	3	661	2	C-109	08/10/90	07/17/90	24	948
1	C-108	31-Oct-89	27-Oct-89	4	665	2	C-109	08/23/90	08/10/90	13	961
1	C-108	24-Jan-90	31-Oct-89	85	750	2	C-109	10/09/90	08/23/90	47	1008
1	C-108	16-Feb-90	24-Jan-90	23	773	2	C-109	10/18/90	10/09/90	9	1017
1	C-108	09-Apr-90	16-Feb-90	52	825	2	C-109	10/25/90	10/18/90	7	1024
1	C-108	25-Apr-90	09-Apr-90	16	841	2	C-109	11/09/90	10/25/90	15	1039
1	C-108	30-Apr-90	25-Apr-90	5	846	2	C-109	11/19/90	11/09/90	10	1049

TABLE 11: WEIBULL PARAMETERS OF MAJOR COMPONENTS OF REPAIRABLE SYSTEM IV

COMPONENT	SHAPE (β_W)	CHARACTERISTIC LIFE (η) (Days)	t_0
1	0.1226	186,904.00	40.99
2	0.9439	100.07	.12
3	0.7128	1,831.76	-0.69
4	1.3833	174.20	-13.78

TABLE 12: FAILURE PREDICTIONS BY MONTE CARLO SIMULATION FOR MAJOR COMPONENTS OF REPAIRABLE SYSTEM IV

COMPONENT	NUMBER OF FAILURES PREDICTED FOR	
	One Year	Two Years
1	2.7	5.3
2	3.8	7.6
3	.6	1.1
4	1.7	3.4
Total System	8.8	17.5

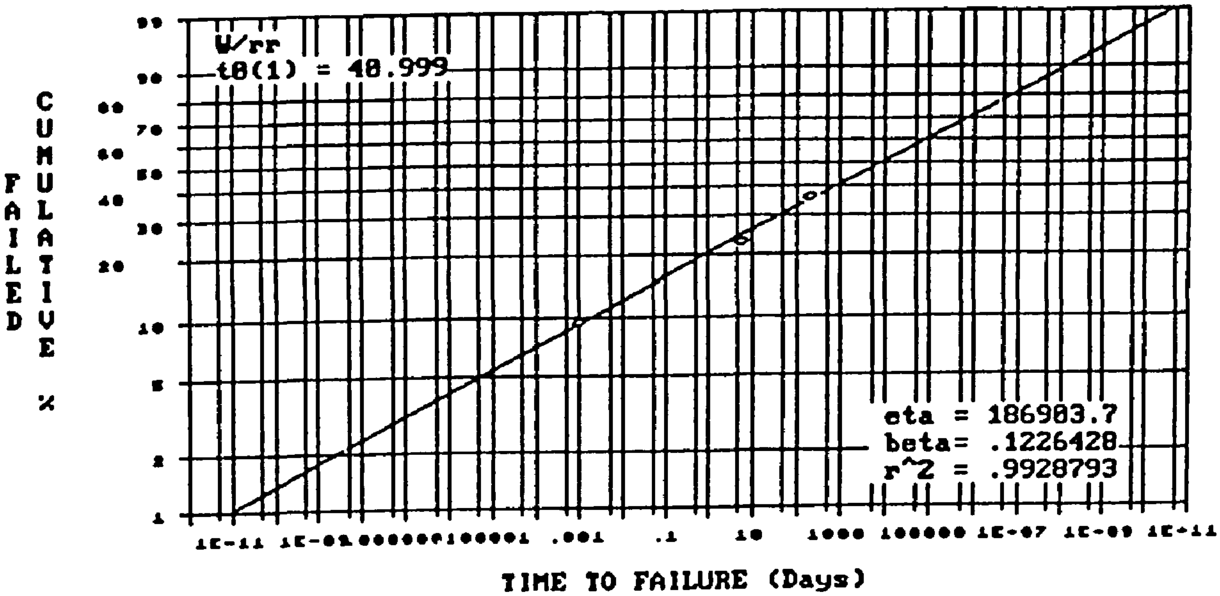


FIGURE 20: WEIBULL ANALYSIS FOR COMPONENT 1 SYSTEM IV

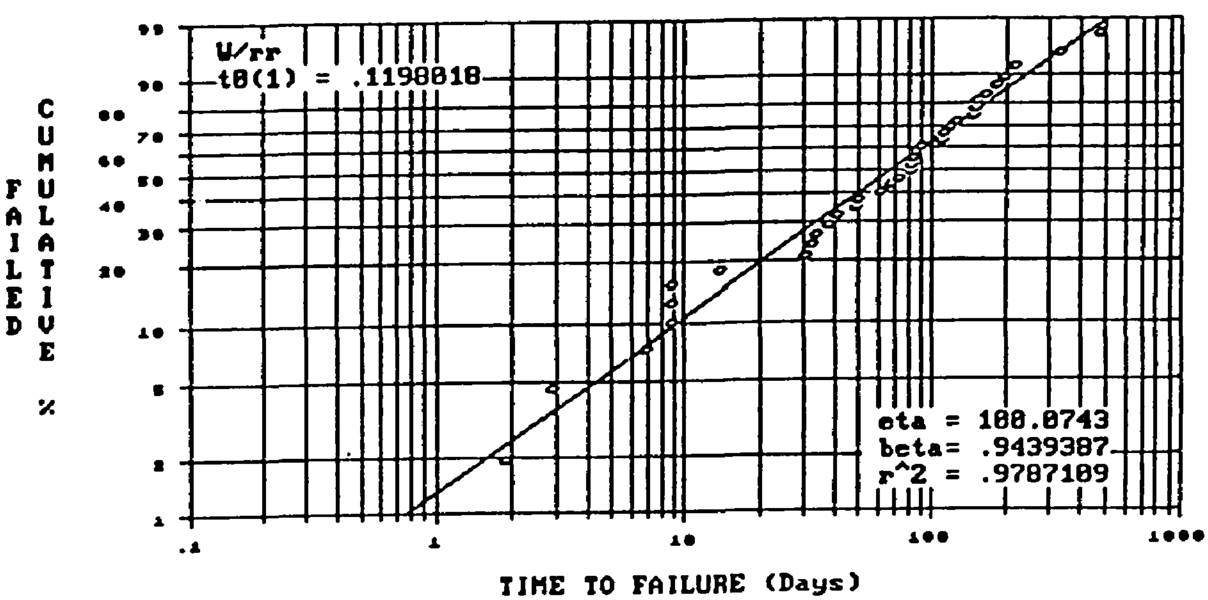


FIGURE 21: WEIBULL ANALYSIS FOR COMPONENT 2 SYSTEM IV

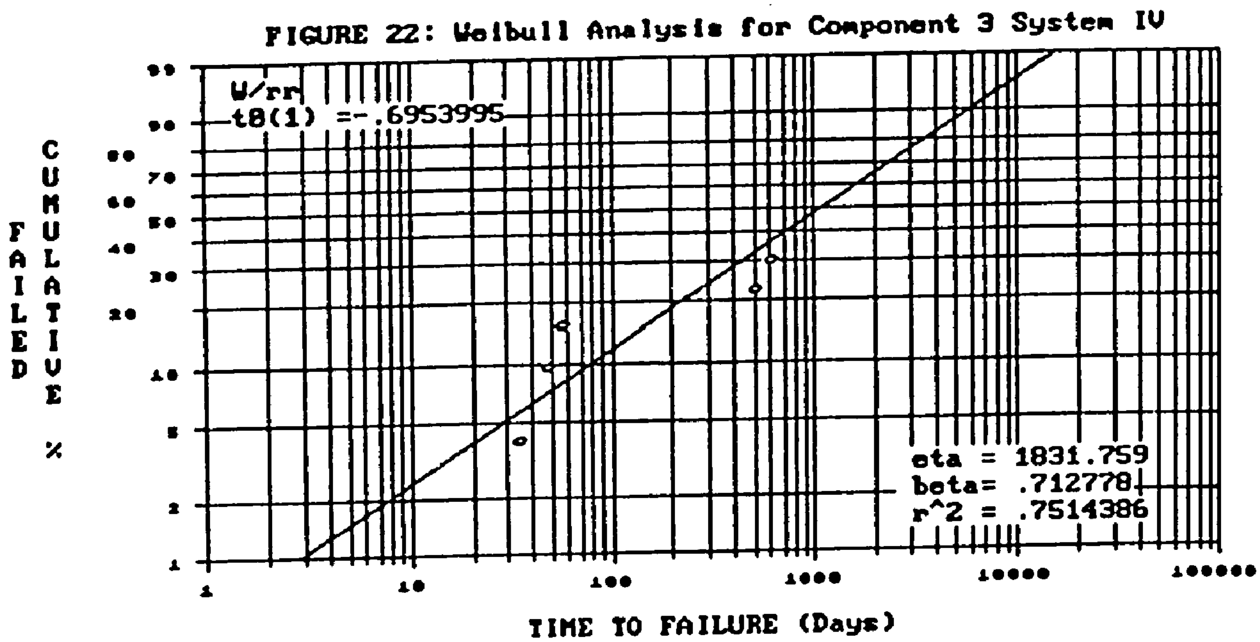


FIGURE 22: WEIBULL ANALYSIS FOR COMPONENT 3 SYSTEM IV

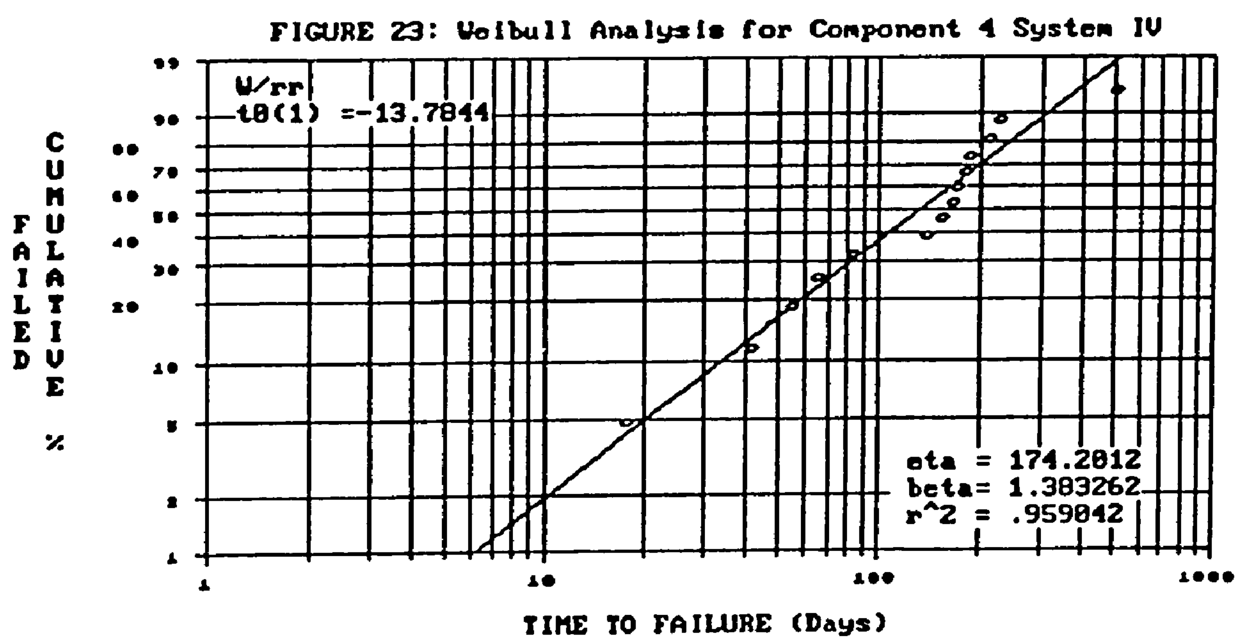


FIGURE 23: WEIBULL ANALYSIS FOR COMPONENT 4 SYSTEM IV

CASE 5. REPAIRABLE SYSTEM V DATABASE

The database for Repairable System V (Table 13) consists of two reciprocating compressors that are also two staged. Time zero is set as January 28, 1988 and failure data is available through December 14, 1990.

The iteration program was utilized to arrive at the Crow NHPP estimates of λ and β of $\hat{\lambda}$=.0071705 and $\hat{\beta}$=1.21501.

The Crow NHPP predictions were calculated to be 9.31 failures for one year and 21.6 failures for two years.

Failure data for five different components were fitted to Weibull distributions and the parameters are given in Table 14. The Weibull charts are given in Figures 24 through 28.

The simulation predictions were 8.1 failures for one year and 15.3 failures for two years. The individual component contributions are given in Table 15.

TABLE 13: REPAIRABLE SYSTEMS DATABASE V

SYSTEM	EQUIP ID	DATE	LAST REPAIRED	DAYS BETW	CUM.
1	C-501	28-Jan-88	28-Jan-88	0	0
1	C-501	17-Feb-88	28-Jan-88	20	20
1	C-501	16-Mar-88	17-Feb-88	28	48
1	C-501	21-Mar-88	16-Mar-88	5	53
1	C-501	23-Mar-88	21-Mar-88	2	55
1	C-501	8-Apr-88	23-Mar-88	16	71
1	C-501	15-Jul-88	8-Apr-88	98	169
1	C-501	23-Aug-88	15-Jul-88	39	208
1	C-501	7-Sep-88	23-Aug-88	15	223
1	C-501	9-Sep-88	7-Sep-88	2	225
1	C-501	6-Jan-89	9-Sep-88	119	344
1	C-501	27-Feb-89	6-Jan-89	52	396
1	C-501	21-Apr-89	27-Feb-89	53	449
1	C-501	10-May-89	21-Apr-89	19	468
1	C-501	7-Jun-89	10-May-89	28	496
1	C-501	13-Jun-89	7-Jun-89	6	502
1	C-501	28-Jun-89	13-Jun-89	15	517
1	C-501	8-Sep-89	28-Jun-89	72	589
1	C-501	21-Sep-89	8-Sep-89	13	602
1	C-501	28-Sep-89	21-Sep-89	7	609
1	C-501	4-Oct-89	28-Sep-89	6	615
1	C-501	9-Oct-89	4-Oct-89	5	620
1	C-501	24-Oct-89	9-Oct-89	15	635
1	C-501	2-Jan-90	24-Oct-89	70	705
1	C-501	3-Jan-90	2-Jan-90	1	706
1	C-501	4-Jan-90	3-Jan-90	1	707
1	C-501	22-Jan-90	4-Jan-90	18	725
1	C-501	25-Jan-90	22-Jan-90	3	728
1	C-501	6-Feb-90	25-Jan-90	12	740
1	C-501	6-Mar-90	6-Feb-90	28	768
1	C-501	7-Mar-90	6-Mar-90	1	769
1	C-501	14-Mar-90	7-Mar-90	7	776
1	C-501	15-Mar-90	14-Mar-90	1	777
1	C-501	20-Mar-90	15-Mar-90	5	782
1	C-501	3-Apr-90	20-Mar-90	14	796
1	C-501	11-Apr-90	3-Apr-90	8	804
1	C-501	17-Apr-90	11-Apr-90	6	810
1	C-501	19-Apr-90	17-Apr-90	2	812
1	C-501	7-May-90	19-Apr-90	18	830
1	C-501	16-May-90	7-May-90	9	839
1	C-501	21-May-90	16-May-90	5	844
1	C-501	22-May-90	21-May-90	1	845
1	C-501	19-Jun-90	22-May-90	28	873
1	C-501	21-Jun-90	19-Jun-90	2	875
1	C-501	19-Nov-90	21-Jun-90	151	1026
1	C-501	20-Nov-90	19-Nov-90	1	1027
1	C-501	14-Dec-90	20-Nov-90	24	1051
2	C-502	17-Feb-88	28-Jan-88	20	20
2	C-502	14-Apr-88	17-Feb-88	57	77
2	C-502	15-Apr-88	14-Apr-88	1	78
2	C-502	17-Aug-88	15-Apr-88	124	202
2	C-502	21-Oct-88	17-Aug-88	65	267
2	C-502	25-Oct-88	21-Oct-88	4	271
2	C-502	2-Feb-89	25-Oct-88	100	371
2	C-502	15-Mar-89	2-Feb-89	41	412
2	C-502	30-May-89	15-Mar-89	76	488
2	C-502	22-Jun-89	30-May-89	23	511
2	C-502	9-Jul-89	22-Jun-89	17	528
2	C-502	28-Jul-89	9-Jul-89	19	547
2	C-502	15-Feb-90	28-Jul-89	202	749
2	C-502	6-Mar-90	15-Feb-90	19	768
2	C-502	3-Apr-90	6-Mar-90	28	796
2	C-502	19-Apr-90	3-Apr-90	16	812
2	C-502	10-May-90	19-Apr-90	21	833
2	C-502	19-Jun-90	10-May-90	40	873
2	C-502	3-Jul-90	19-Jun-90	14	887
2	C-502	18-Jul-90	3-Jul-90	15	902
2	C-502	1-Aug-90	18-Jul-90	14	916
2	C-502	14-Dec-90	1-Aug-90	135	1051

TABLE 14: WEIBULL PARAMETERS OF MAJOR COMPONENTS OF REPAIRABLE SYSTEM V

COMPONENT	SHAPE (β_W)	CHARACTERISTIC LIFE (η) (Days)	t_0
1	0.5105	549.28	51.04
2	0.6979	259.00	2.16
3	1.1708	764.96	47.72
4	1.5648	194.72	-22.16
5	0.2534	5000.31	30.99

TABLE 15: FAILURE PREDICTIONS BY MONTE CARLO SIMULATION FOR MAJOR COMPONENTS OF REPAIRABLE SYSTEM V

COMPONENT	NUMBER OF FAILURES PREDICTED FOR	
	One Year	Two Years
1	1.7	3.3
2	2.0	3.9
3	.4	.7
4	1.8	3.3
5	2.2	4.1
Total System	8.1	15.3

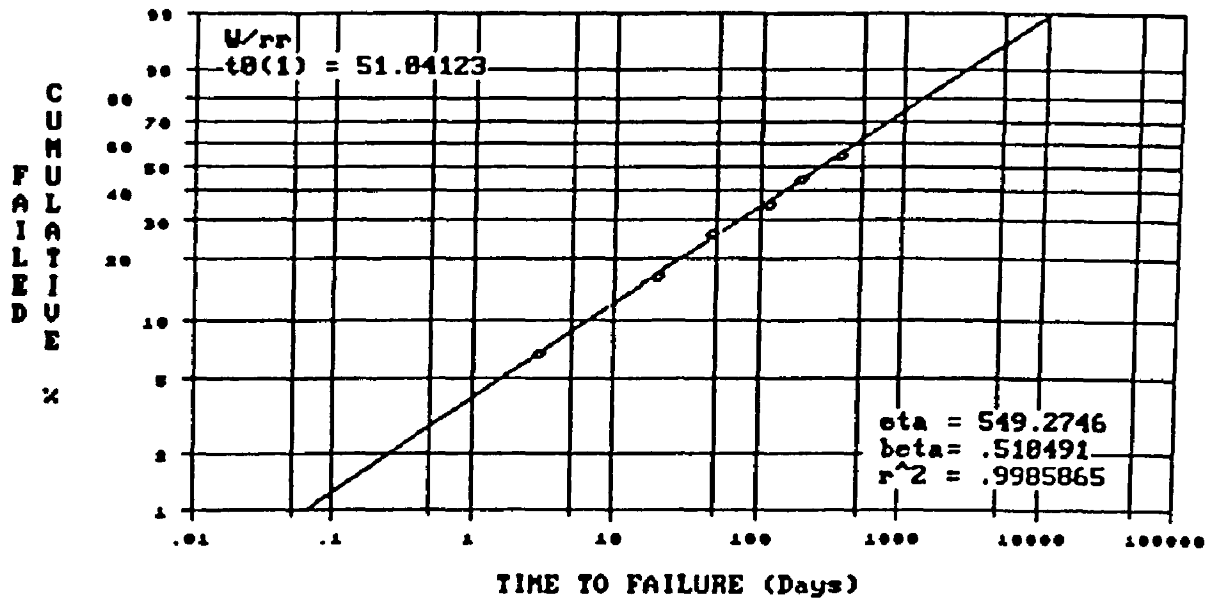


FIGURE 24: WEIBULL ANALYSIS FOR COMPONENT 1 SYSTEM V

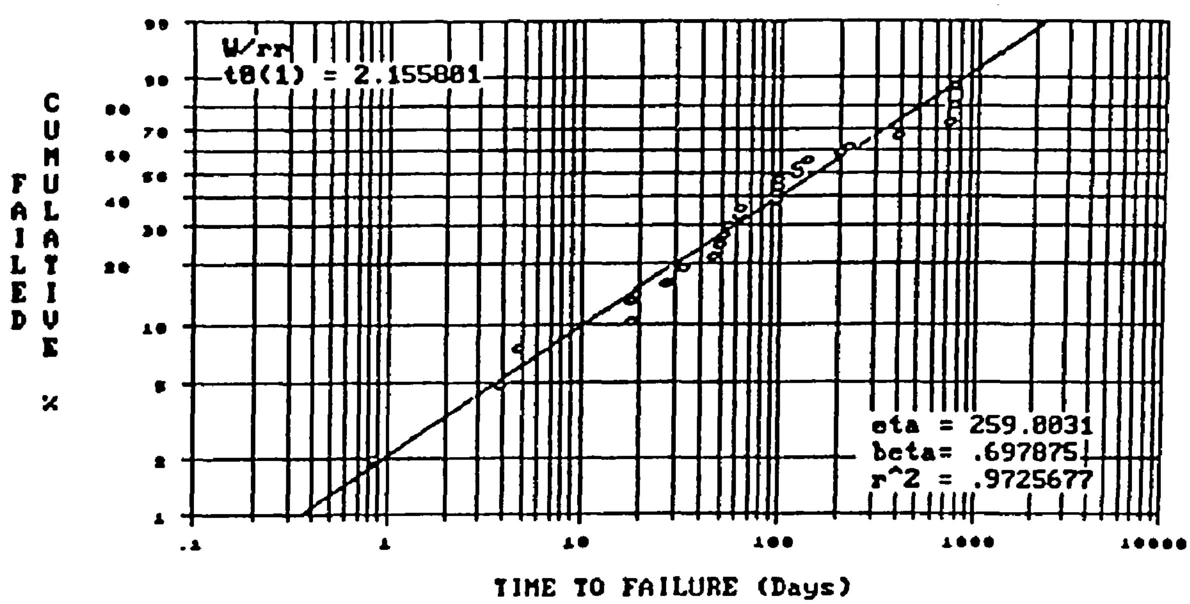


FIGURE 25: WEIBULL ANALYSIS FOR COMPONENT 2 SYSTEM V

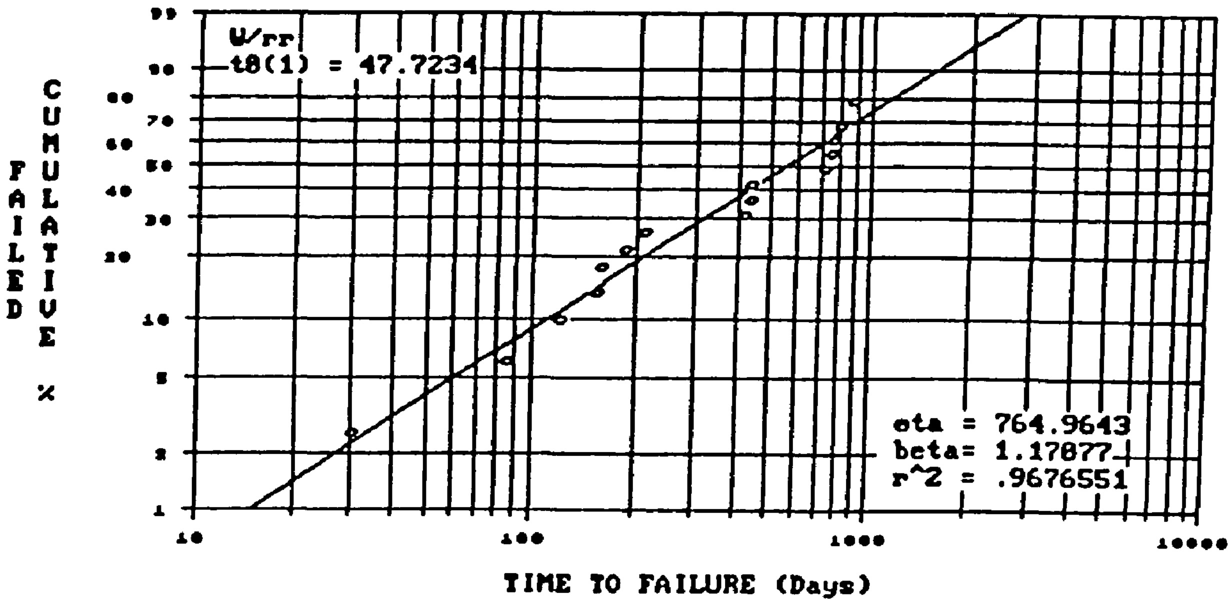


FIGURE 26: WEIBULL ANALYSIS FOR COMPONENT 3 SYSTEM V

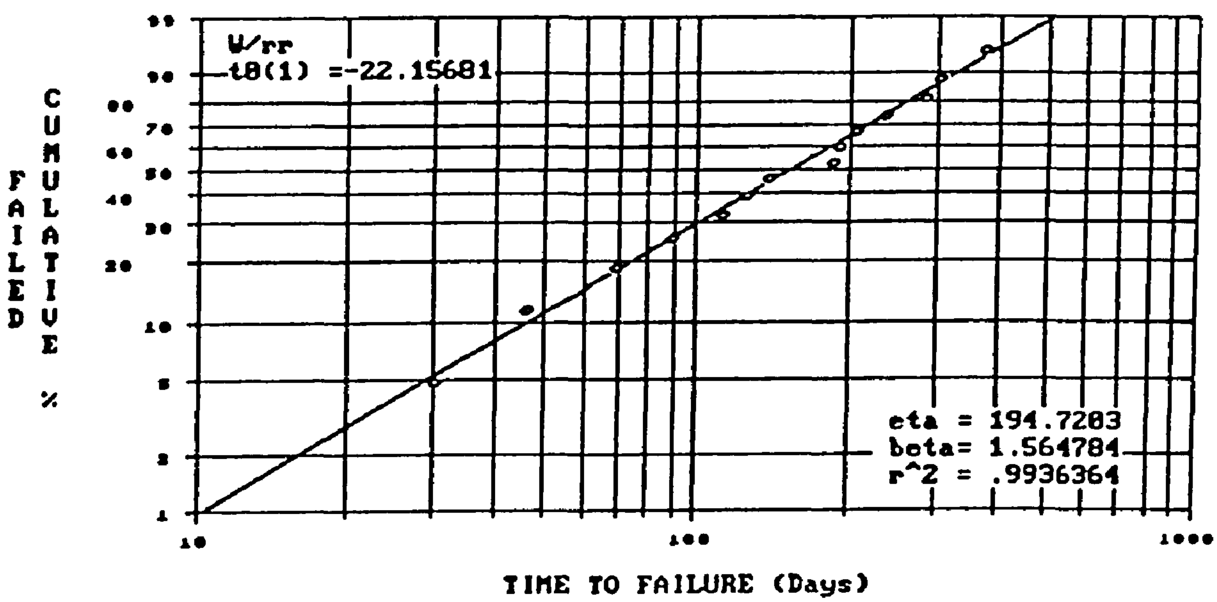


FIGURE 27: WEIBULL ANALYSIS FOR COMPONENT 4 SYSTEM V

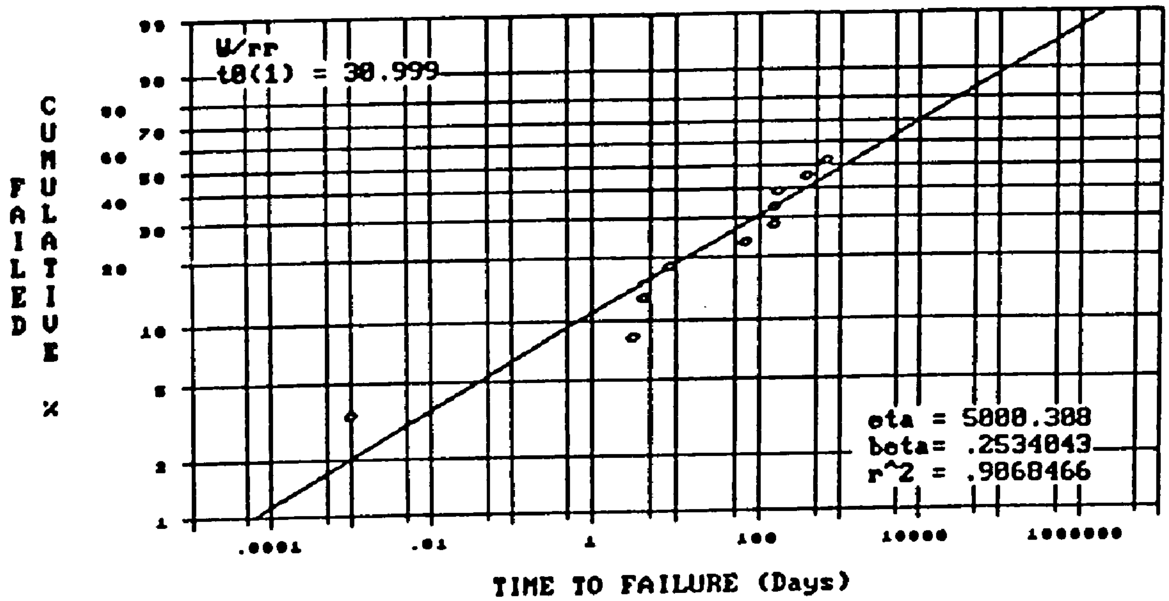


FIGURE 28: WEIBULL ANALYSIS FOR COMPONENT 5 SYSTEM V

CASE 6. REPAIRABLE SYSTEM VI DATABASE

The database for Repairable System VI (Table 16) consists of two single stage reciprocating compressors. Time zero is set as September 18, 1987 and failure data is available through November 8, 1990.

The iteration program was used to arrive at the Crow NHPP estimates of λ and β of $\hat{\lambda}$ =.0029688 and $\hat{\beta}$=1.268006. The Crow NHPP predictions were calculated to be 5.27 failures for one year and 12.69 failures for two years.

Failure data for three components were fitted to Weibull distributions and the parameters are listed in Table 17. The Weibull charts are shown in Figures 29 through 31.

The Monte Carlo simulation predictions were 5.4 failures for one year and 10.4 failures for two years. The number of failures predicted for each component is given in Table 18.

TABLE 16: REPAIRABLE SYSTEM DATABASE VI

SYSTEM	EQUIP ID	DATE	LAST REPAIRED	DAYS BETW	CUM.
1	C-106	14-Nov-87	18-Sep-87	57	57
1	C-106	10-Mar-88	14-Nov-87	117	174
1	C-106	2-May-88	10-Mar-88	53	227
1	C-106	30-Jun-88	2-May-88	59	286
1	C-106	8-Jul-88	30-Jun-88	8	294
1	C-106	22-Jul-88	8-Jul-88	14	308
1	C-106	25-Jul-88	22-Jul-88	3	311
1	C-106	26-Jul-88	25-Jul-88	1	312
1	C-106	28-Jul-88	26-Jul-88	2	314
1	C-106	7-Sep-88	28-Jul-88	41	355
1	C-106	11-Oct-88	7-Sep-88	34	389
1	C-106	5-Apr-89	11-Oct-88	176	565
1	C-106	15-May-89	5-Apr-89	40	605
1	C-106	21-Jun-89	15-May-89	37	642
1	C-106	30-Jun-89	21-Jun-89	9	651
1	C-106	16-Aug-89	30-Jun-89	47	698
1	C-106	26-Oct-89	16-Aug-89	71	769
1	C-106	31-Oct-89	26-Oct-89	5	774
1	C-106	29-Jun-90	31-Oct-89	241	1015
1	C-106	16-Aug-90	29-Jun-90	48	1063
1	C-106	29-Aug-90	16-Aug-90	13	1076
1	C-106	9-Oct-90	29-Aug-90	41	1117
1	C-106	8-Nov-90	9-Oct-90	30	1147
2	C-107	18-Sep-87	18-Sep-87	0	0
2	C-107	7-Jan-88	18-Sep-87	111	111
2	C-107	29-Jan-88	7-Jan-88	22	133
2	C-107	26-Apr-88	29-Jan-88	88	221
2	C-107	4-May-88	26-Apr-88	8	229
2	C-107	1-Jul-88	4-May-88	58	287
2	C-107	10-Oct-88	1-Jul-88	101	388
2	C-107	1-Feb-89	10-Oct-88	114	502
2	C-107	9-Feb-89	1-Feb-89	8	510
2	C-107	19-Apr-89	9-Feb-89	69	579
2	C-107	6-Sep-89	19-Apr-89	140	719
2	C-107	27-Sep-89	6-Sep-89	21	740
2	C-107	14-Nov-89	27-Sep-89	48	788
2	C-107	17-Nov-89	14-Nov-89	3	791
2	C-107	1-Feb-90	17-Nov-89	76	867
2	C-107	16-Feb-90	1-Feb-90	15	882
2	C-107	6-Mar-90	16-Feb-90	18	900
2	C-107	21-Mar-90	6-Mar-90	15	915
2	C-107	23-Mar-90	21-Mar-90	2	917
2	C-107	18-Jun-90	23-Mar-90	87	1004

TABLE 17: WEIBULL PARAMETERS OF MAJOR COMPONENTS OF REPAIRABLE SYSTEM VI

COMPONENT	SHAPE (β_W)	CHARACTERISTIC LIFE (η) (Days)	t_0
1	0.8267	402.43	10.56
2	0.5271	310.20	97.01
3	0.5127	247.53	10.73

TABLE 18: FAILURE PREDICTIONS BY MONTE CARLO SIMULATION FOR MAJOR COMPONENTS OF REPAIRABLE SYSTEM VI

COMPONENT	NUMBER OF FAILURES PREDICTED FOR	
	One Year	Two Years
1	1.1	2.1
2	1.9	3.7
3	2.4	4.5
Total System	5.4	10.4

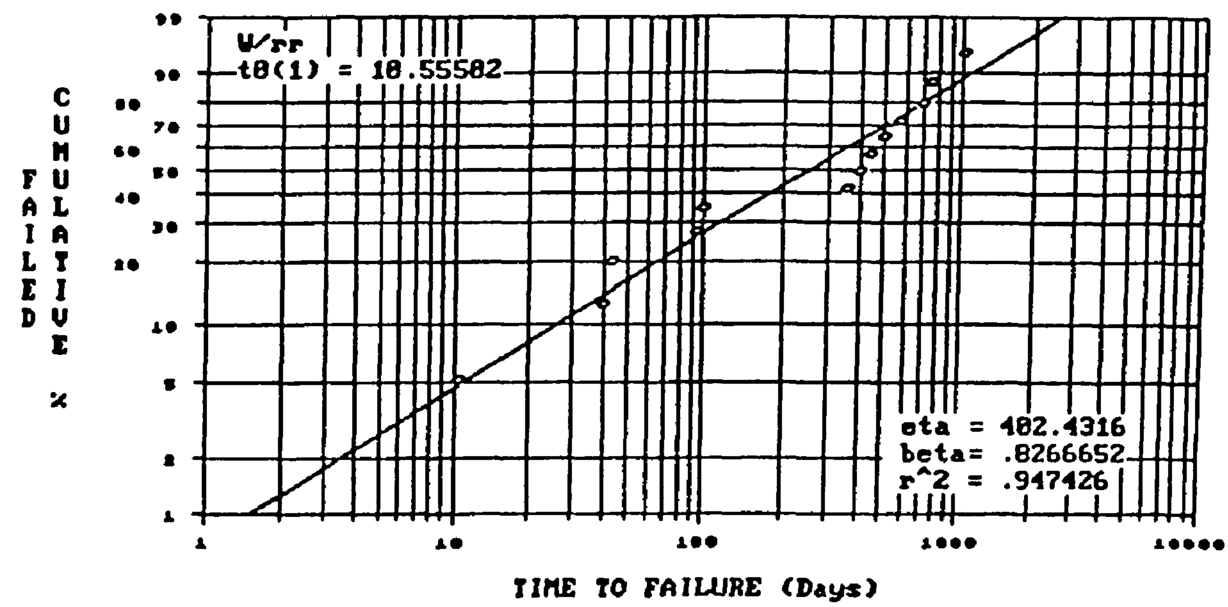


FIGURE 29: WEIBULL ANALYSIS FOR COMPONENT 1 SYSTEM VI

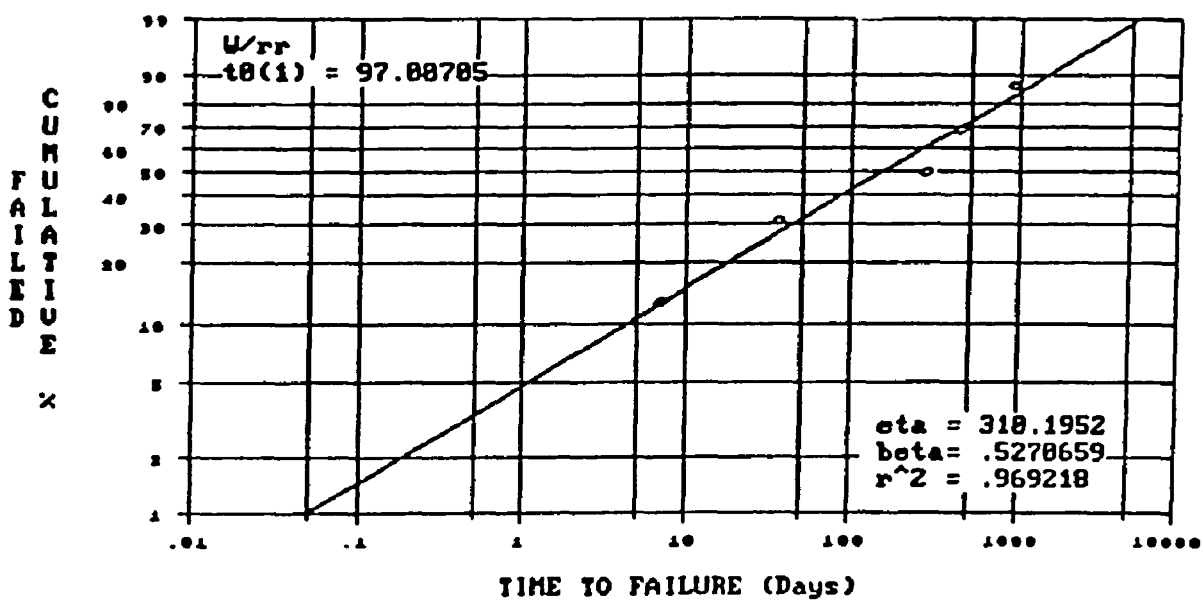


FIGURE 30: WEIBULL ANALYSIS FOR COMPONENT 2 SYSTEM VI

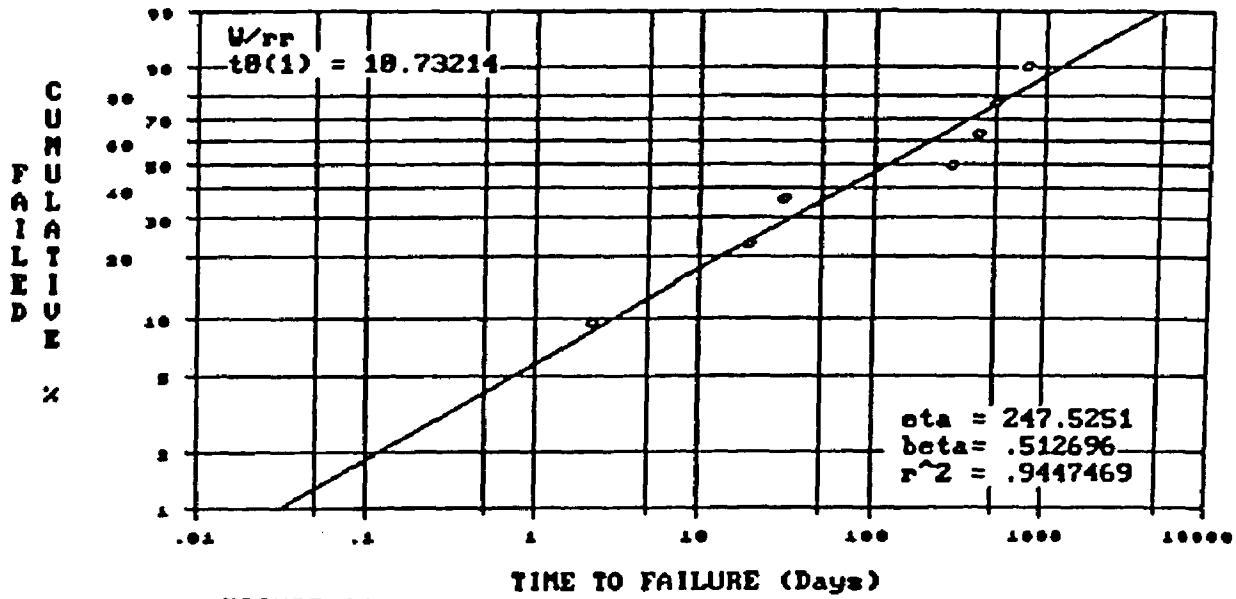


FIGURE 31: WEIBULL ANALYSIS FOR COMPONENT 3 SYSTEM VI

EXPLANATION OF THE MONTE CARLO SIMULATION

In each of the six cases a Monte Carlo simulation based on the Weibull distribution is used to predict the number of failures expected for each system. Some explanation of the technique and its application to the data are in order.

The derivation of the equation used in the Monte Carlo simulation is based on each component exhibiting a continuous distribution of times-to-failure that fits a Weibull distribution.

The simulation is based on the Weibull distribution which is defined mathematically as follows:

$$F(t) = 1 - e^{-(\frac{(t-t_o)}{\eta})^{\beta_w}} \quad (3.9)$$

where,

$F(t)$ = fraction failing (the cumulative fraction of a group of identical components that will fail by time, t),

t = failure time,

t_o = starting point or origin of distribution,

η = characteristic life or scale parameter,

β_w = slope or shape parameter, and,

e = exponential.

The fraction of components which have not failed up to time, t, is [1-F(t)]. This is often called reliability at time, t, and is denoted by R(t).

Rearranging the distribution function:

$$1 - F(t) = e^{-\left(\frac{t-t_o}{\eta}\right)^{\beta_w}} \quad , \tag{3.10}$$

and let $t_o = 0$

then,
$$1 - F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta_w}} \quad , \tag{3.11}$$

and,

$$1 - F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta_w}} \quad , \tag{3.12}$$

$$\frac{1}{1 - F(t)} = e^{\left(\frac{t}{\eta}\right)^{\beta_w}} \quad \text{, and,} \tag{3.13}$$

$$\ln\left(\frac{1}{1 - F(t)}\right) = \left(\frac{t}{\eta}\right)^{\beta_w} \quad . \tag{3.14}$$

Solving for t, failure time,

$$\left(\frac{t}{\eta}\right)^{\beta_w} = \ln\left(\frac{1}{1 - F(t)}\right) \quad , \tag{3.15}$$

$$\frac{t}{\eta} = \left[\ln\left(\frac{1}{1 - F(t)}\right)\right]^{1/\beta_w} \quad \text{, and,} \tag{3.16}$$

$$t = \eta\left[\ln\left(\frac{1}{1 - F(t)}\right)\right]^{1/\beta_w} \quad . \tag{3.17}$$

This equation is used in the Monte Carlo simulation in the following form:

$$time-to-failure = \eta\left[\ln\left(\frac{1}{1-random\ \ number}\right)\right]^{1/\beta_w} \qquad (3.18)$$

where the random numbers are uniformly distributed between 0 and 1 and the η and β_W are the Weibull parameters of the specific component failure mode.

The following steps are then carried out by the simulation as shown in Figure 32:

STEP 1:

Generate random times to failure for each component failure mode.

STEP 2:

Find the minimum of the times-to-failure of the different component failure modes.

STEP 3:

Compare this failure time to the scheduled inspection time. (For this study, this was entered as a constant and kept very large in all cases so that simulation would be for continuous operation.)

STEP 4:

Record the cause of the failure (which component).

STEP 5:

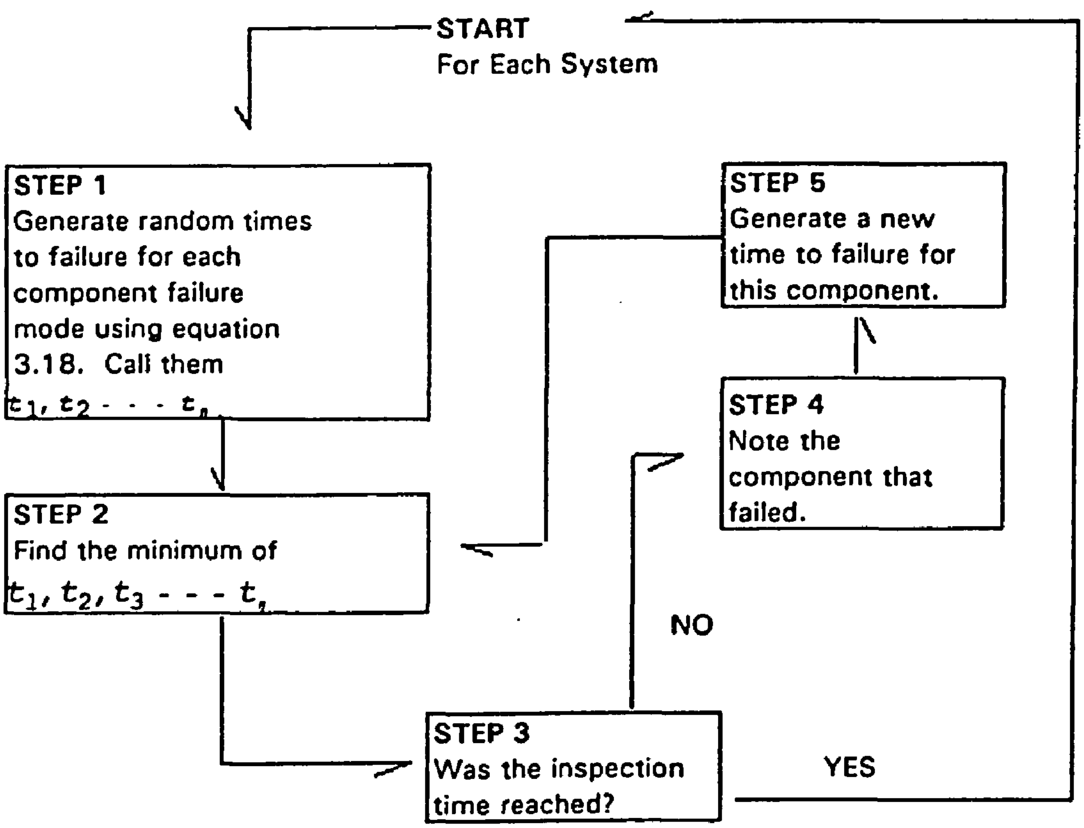
Generate a new time-to-failure for this component from the recent failure time and proceed to Step 2 again to compare for the minimum time-to-failure of the components calculated.

INPUT

(1) Weibull Component Failure Distribution Parameters (η, β_w)

(2) Usage/Month

(3) Scheduled Inspection Time



OUTPUT

Average cumulative failures over the time period specified.

FIGURE 32: MONTE CARLO SIMULATION

The inputs for this simulation program are the Weibull failure distribution parameters (η, β_W) for the components as well as a usage per month and the scheduled inspection or overhaul time.

The output is the average cumulative additional failures over the time period specified. The number of failures is broken down by the specific components used in the simulation.

Assumptions:

1. Only the failed components are replaced.
2. At failure or scheduled inspection the components are made "good-as-new".

COMPARISON OF RESULTS

In Table 19 the failure predictions for periods of one and two years are compared for the six repairable systems under study. A paired t-test was utilized to compare the results generated by the two different methods. The difference was determined in each of the twelve cases and the average or mean of these differences, $\bar{x}$, was calculated as 1.4461. The sample standard deviation, s, was calculated to be 2.783536 and then from the equation for the standard deviation of the mean

$$s_{\bar{x}} = \frac{s}{\sqrt{\eta}} \tag{3.19}$$

where $\eta =$ the number of samples,

the result $s_{\bar{x}}=0.803536$ was determined.

TABLE 19: STATISTICAL COMPARISON OF PREDICTED FAILURES FROM CROW MODEL AND MONTE CARLO SIMULATION

SYSTEM	CROW MODEL	SIMULATION	DIFFERENCE
		(1 YEAR)	(CROW-SIM)
I	0.555	0.6	-0.045
II	2.374	2.6	-0.226
III	4.825	5.1	-0.275
IV	10.206	8.8	1.406
V	9.306	8.1	1.206
VI	5.267	5.4	-0.133
		(2 YEARS)	
I	1.48	1.6	-0.12
II	4.47	6.0	-1.53
III	11.22	10.4	0.82
IV	25.17	17.5	7.67
V	21.60	15.3	6.3
VI	12.68	10.4	2.28

$\bar{x} = 1.4461$

$s = 2.783536$

$$s_{\bar{x}} = \frac{s}{\sqrt{\eta}} = 0.803536$$

$$t = \frac{\bar{x}}{s_{\bar{x}}} = 1.7996668$$

$t_{.05,11} = 2.201$

The Student's t value given by

$$t = \frac{\bar{x}}{s_{\bar{x}}} \tag{3.20}$$

was determined to be $t=1.79967$.

From the table for t-distributions (18) t with 95 percent confidence and 11 degrees of freedom has a value of $t_{.05,11}=2.2010$ and since the value of t calculated , 1.79967, is less than $t_{.05,11}=2.2010$ the paired t-test shows no significant difference in the two methods.

From this, one can conclude that there is no appreciable difference in the population means of the two different methods based on the sample evidence; therefore, the simulation technique can be used since it is far more efficient than Crow's model.

System I is a repairable system that has a well defined number of mechanical components and a well documented failure history. The results obtained from Crow's model and the simulation based on Weibull parameters of the four major component failure modes correlate closely.

System II is a vertical compressor system made up of many mechanical components, three of which were chosen due to their failure predominance in the failure database. The simulation model based on the Weibull parameters of these three major component failure modes gave predictions that duplicated, with 95 percent confidence, the predictions for the overall system given by the Crow NHPP model. This reinforces the hypotheses that the three components modelled accounted for most of the variables that need to be addressed from a preventive maintenance basis.

The System III database consists of failure data from five single stage reciprocating compressors of identical design and manufacture. The Crow NHPP model indicates that one of these compressors is down 4.8 times per year and the simulation utilizing four component failure mode Weibull parameters predicts that these four components will cause 5.1 failures for the same time frame. Here again, the simulation has the appropriate Weibull parameters included to match the NHPP model.

System IV represents two reciprocating compressors that are two staged and are in identical service. The simulation of the four major component failure modes in this case predicts lower incidents than the Crow NHPP model. The results indicate that these type compressors will fail about ten times per year and that nine out of the ten times one of the four major components will be the cause of the failure.

System V contains two-staged compressors of the same design and manufacture as System IV but in different service. The results were similar to System IV. In System V's case it took Weibull data for five components to obtain the simulation that predicts 8.1 failures for a year compared to 9.3 failures by the NHPP model.

System VI consists of two single stage compressors of identical design and in the same service. The simulation required three components in this case to closely match the NHPP model predictions.

CROW'S MODEL PARAMETERS OF THE TOTAL SYSTEMS

The Crow model maximum likelihood estimates for λ and β are listed in Table 20 along with the corresponding expected number of failures of the respective systems. All but one of the repairable systems studied had β values greater than one. For all $\beta > 1$, the failure intensity function, $u\ (\ t\ )$ is increasing and the intervals between successive failures are stochastically decreasing. This is characteristic of a wearout situation. System II has a β value of less than one and $u\ (\ t\ )$ is decreasing. This is characteristic of an improving situation where the intervals between successive failures are stochastically increasing.

WEIBULL PARAMETERS OF COMPONENTS

Table 21 lists the individual components and their respective Weibull parameters. The parameters listed are the shape parameter or slope of the Weibull curve, β_W, and the scale parameter, η , or characteristic life. The components with $\beta_W < 1$ are indicative of infant mortality type failures where the failure rate is decreasing with age. The components with $\beta_W > 1$ are indicative of wear out failures where the failure rate is increasing with age. A component with $\beta_W = 1$ would be representative of a random or constant failure rate.

Out of the 23 components modelled by the Weibull distribution there are 12 with $\beta_W > 1$ and 11 with $\beta_W < 1$.

TABLE 20: CROW MODEL NHPP PARAMETERS

SYSTEM	MAX. LIKELIHOOD ESTIMATES		FAILURES PREDICTED	
	$\hat{\lambda}$	$\hat{\beta}$	$E[N(t)] = \lambda t^{\beta}$	
			t=365	t=730
I	0.00013448	1.411001	0.555	1.475
II	0.0109319	0.91199	2.374	4.70
III	0.0036529	1.218006	4.825	11.220
IV	0.0047069	1.30201	10.206	25.166
V	0.0071705	1.21501	9.306	21.603
VI	0.0029688	1.268006	5.267	12.685

TABLE 21: WEIBULL PARAMETERS OF MAJOR COMPONENTS

SYSTEM	COMPONENT	SHAPE (β_W)	CHAR LIFE (DAYS)	FAILURE PREDICTION 1 YR	2 YR
I	1	2.028	768	.1	.3
I	2	3.183	393	.3	1.0
I	3	1.859	1366	.1	.1
I	4	3.762	806	.0	.1
II	1	1.183	264	1.1	2.5
II	2	1.968	344	.4	1.1
II	3	2.045	214	1.1	2.3
III	1	1.265	210	1.5	3.0
III	2	0.494	599	1.6	3.3
III	3	0.833	248	1.8	3.7
III	4	1.613	702	.2	.4
IV	1	0.1226	186944	2.7	5.3
IV	2	0.9439	100	3.8	7.6
IV	3	0.7128	1831	.6	1.1
IV	4	1.3833	160	1.7	3.4
V	1	0.5105	600	1.7	3.3
V	2	0.6979	261	2.0	3.9
V	3	1.1708	813	.4	.7
V	4	1.5648	173	1.8	3.3
V	5	0.2534	5031	2.2	4.1
VI	1	0.8267	413	1.1	2.1
VI	2	0.5271	407	1.9	3.7
VI	3	0.5127	258	2.4	4.5

SUMMARY OF WORK

Databases were studied for each of six different system types. The data sample was limited to the following types of critical mechanical equipment in the plastics producing plants in Dow's Louisiana Division:

1. Reactor agitator systems - Database I
2. Centrifugal Compressors - Database II
3. Single Stage Reciprocating Compressors - Databases III and VI
4. Two-staged Reciprocating Compressors - Databases IV and V

The failure data for each system database was analyzed by the Crow NHPP model and failure predictions for the system as a whole were generated in each case. The expected number of failures predicted for the respective system by the Crow model is considered the standard for such a system and attempts were made to match that prediction using a Monte Carlo simulation that utilizes Weibull parameters of the major components of the system.

The objective was to prove that a simulation based on Weibull parameters of the major component failure modes is able to duplicate the overall system prediction that the Crow NHPP model gives.

Another objective of the research was to demonstrate that continuous distribution functions such as the Weibull can be used to model component failure data or hazard functions but are not appropriate for repairable multi-component systems.

The use of the Monte Carlo simulation approach allows the continuous distributions of the major component failure modes to be utilized to model the overall system performance. The objective was to prove that this is a viable technique and more valuable to reliability studies because it allows for the determination of a finite number of parts that contribute to the overall system downtime. This information can be used to design an optimum preventive maintenance program or to guide research into more reliable components or parts.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

Conclusions:

There is a right way and a wrong way to analyze the failure data for repairable systems. Ascher and Feingold (2) cited several examples where repairable systems failure data were incorrectly analyzed. Often, this occurred because the data were analyzed using techniques which would be suitable for components or parts such as fitting the failure data to a Weibull distribution. Under the power law nonhomogeneous Poisson process, NHPP, time to first failure is Weibull distributed but after the first failure, successive times between failures are neither independent nor identically distributed.

This research has utilized Crow's nonhomogeneous Poisson process (7) to model multi-component mechanical systems. The NHPP is a stochastic point process or mathematical model for a physical phenomenon characterized by highly localized events distributed randomly in a continuum. The continuum is time and the highly localized events are failures which occur at instants within the time continuum. The failure data for mechanical systems in this research were modelled by Crow's NHPP model, and the parameters, λ and β, of the stochastic process are given in Table 20 along with the predicted number of failures for periods of one and two years.

From results such as those in Table 20, one can determine whether the reliability of the system in question is improving or deteriorating. For values of the parameter, $\beta > 1$, the failure intensity function for the system as a whole is increasing and the intervals between successive failures are stochastically decreasing. This is characteristic of a wearout situation. Systems I, III, IV, V, and VI all exhibit $\beta > 1$, and therefore deterioriation in reliability.

System II has a β value of less than one and the failure intensity function for the system as a whole is decreasing. The intervals between successive failures are stochastically increasing and this is characteristic of an improving situation.

Although the results of the Crow NHPP models are statistically correct and give accurate "failure rate" data of the systems as a whole, they are of limited value in determining how to improve the reliability of the systems. The Crow model does not tell one what to work on to improve the reliability of the system.

The author has succeeded in this research in improving the Crow NHPP results by demonstrating that the Crow NHPP results can be utilized as a standard or guide for a simulation model in each case based on the appropriate individual components of the respective system.

The individual component failure data were fitted to Weibull distributions and the resulting distribution function parameters are listed in Table 21. The parameters listed are the shape parameter or slope of the Weibull curve, β_W, and the scale parameter, η, or characteristic life.

For each system, a Monte Carlo simulation utilizing the individual component Weibull parameters was able to duplicate, with 95% confidence, the overall system predictions that the Crow NHPP model gave. This technique of using the Crow NHPP model to serve as a gauge to determine when the simulation has the appropriate component failure modes included was proven viable by this research.

The two methods of failure prediction are compared in Table 19. A paired *t*-test was utilized to demonstrate that there is no appreciable difference in the population means of the two different methods.

The technique of utilizing the Crow NHPP model, which uses overall system data, to serve as a guide for testing the accuracy and completeness of a Monte Carlo simulation based on Weibull parameters of individual component failure modes, has been proven by the six different cases in this research to be a new and valuable addition to the knowledge base of Maintenance and Reliability Engineering.

The information on the major component failure modes for the six systems in this research can be utilized to design optimum preventive or predictive maintenance programs or to guide research into more reliable components or parts. For example, from Table 21, it can be seen that for System I, Component 2 is the major contributor to predicted failures. It also can be seen to have the lowest characteristic life, η, of the four components involved and its shape parameter, β_W, is equal to 3.183 which indicates that it is in wearout region of the bathtube curve. This information on Component 2 of System I should prompt research into a better design or better materials of construction of this particular component. Replacement of Component 2 with a more reliable component should be given priority over improvements to the other three components since their characteristic lives are much longer.

This research also points to the importance of accurate, detailed failure records on all components and their specific locations in the systems involved. Equipment records should ideally consist of a time line of all events occurring to the equipment from the design stage right up to its final retirement. A work order system that is equipment based can provide the information needed for reliability studies, but it is important that the disciplined accurate recording of results be consistently enforced.

Recommendations for Future Research:

Based on the results of this research, the author has the following recommendations for future research:

1. The same type analyses should be conducted on different types of equipment or systems other than the types included in this reasearch. Examples might be polymer extruders, pneumatic blowers, ANSI pumps or other chemical process equipment.
2. A simulation should be developed for the case where total rebuilds are done on the systems instead of only the replacement of the failed component.
3. Crow's model should be looked at with the assumption that there is improvement to the system with repair rather than the assumption that each repair returns the system to a condition of "as bad as old" or as "good as new". This would question the requirement of the NHPP model of the failures or events being independent and non-identically distributed.
4. Crow's model should be tried with other distributions than the Weibull for initial failure such as the log normal distribution or exponential distribution.

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APPENDIX

COMPUTER PROGRAMS

1. Iteration of NHPP Model

The program in BASIC does an iteration of the Crow NHPP model equations (equations 2.14 and 2.15 in the body of the paper) to arrive at maximum likelihood estimates of the Crow parameters λ and β.

Input required is the following:

(1) Number of systems or sets of data

(2) Number of data points per set or system

(3) Starting time for each system

(4) Ending time for each system

(5) Time between failures for each set or system.

The output is the maximum likelihood estimates for λ and β.

The example following is for Repairable Systems Database I (Table 1).

ITERATION OF NHPP MODEL

```
90 DIM X(100,100)
100 K=9:'NUMBER OF SYSTEMS
110 N(1)=8:N(2)=9:N(3)=12:N(4)=10:N(5)=14:N(6)=4:N(7)=1:N(8)=2:N(9)=3:'NUMBER DA
TA POINTS/SET
120 S(1)=25:S(2)=0:S(3)=13:S(4)=40:S(5)=120:S(6)=2517:S(7)=3488:S(8)=3488:S(9)=3
214:'STARTING TIME EACH SYSTEM
130 T(1)=2301:T(2)=3229:T(3)=2840:T(4)=3238:T(5)=2753:T(6)=3766:T(7)=3719:T(8)=3
714:T(9)=3767:'ENDING TIME EACH SYSTEM
140 BETA=1.4:'STARTING BETA
150 FOR I = 1 TO K:FOR J = 1 TO N(I):READ X(I,J):NEXT J:NEXT I
160 SUMN=0:SUMTS=0:SUMLTS=0:SUMLXI=0
170 FOR I = 1 TO K
180 SUMN=SUMN+N(I)
190 SUMTS=SUMTS+(T(I)^BETA-S(I)^BETA)
195 IF S(I)=0 THEN SUMLTS=SUMLTS+T(I)^BETA*LOG(T(I)):GOTO 210
200 SUMLTS=SUMLTS+T(I)^BETA*LOG(T(I))-S(I)^BETA*LOG(S(I))
210 FOR J = 1 TO N(I)
220 SUMLXI=SUMLXI+LOG(X(I,J))
230 NEXT J
240 NEXT I
250 LAMBDA=SUMN/SUMTS
260 BETATST=SUMN/(LAMBDA*SUMLTS-SUMLXI)
270 TEST=BETATST-BETA:PRINT BETA,BETATST,LAMBDA
280 IF ABS(TEST)<.0005 GOTO 310:'ALLOWABLE ERROR BETWEEN BETA & BETATST
290 BETA=BETA+.001:'INCREMENT BETA
300 GOTO 160
310 PRINT"BETA=";BETA;"    LAMBDA=";LAMBDA
320 END
1000 DATA 465,626,1105,1368,1694,1950,2217,2301
1010 DATA 308,777,1157,1456,1878,2275,2511,2834,3229
1020 DATA 413,824,1313,1457,1468,1616,2190,2293,2386,2646,2799,2840
1030 DATA 160,822,857,1225,1449,1877,2130,2671,3057,3238
1040 DATA 412,737,1101,1294,1496,1510,1579,1819,1875,1878,2032,2311,2599,2753
1050 DATA 3254,3505,3684,3766
1060 DATA 3719
1070 DATA 3686,3714
1080 DATA 3341,3543,3767

Ok
RUN
 1.4            1.411197       1.470837E-04
 1.401          1.411222       1.458904E-04
 1.402          1.411244       1.44707E-04
 1.403          1.411261       1.435332E-04
 1.404          1.411284       1.423689E-04
 1.405          1.411305       1.412138E-04
 1.406          1.411321       1.400684E-04
 1.407          1.411344       1.389321E-04
 1.408          1.411359       1.378051E-04
 1.409          1.411379       1.366872E-04
 1.410001       1.411398       1.355783E-04
 1.411001       1.411417       1.344785E-04
BETA= 1.411001     LAMBDA= 1.344785E-04
Ok
```

2. Monte Carlo Simulation

The program in FORTRAN is the Monte Carlo simulation described in Figure 32 of the body of the paper.

The β_W and η parameters of the Weibull failure modes are entered as pairs for each component. The usage rate per month and the inspection interval are also required inputs.

The month and year for start of the risk analysis are entered as well as the duration of the risk analysis in years.

The output of the program is the average cumulative failures for each component over the time period specified. A total number of failures for the system as a whole is also output.

```
C
C *********************************************************************
C
C         PROGRAM   : MONTE.FORTRAN
C         PURPOSE   : TO ILLUSTRATE THE MONTE CARLO SIMULATION METHOD
C         REFERENCE : WEIBULL HANDBOOK
C
C
C *********************************************************************
C......VARIABLE DESCRIPTION
C *********************************************************************
C
C       TIME .... ARRAY OF CURRENT POPULATION TIMES (MAX OF 1000)
C
C       USE ..... USAGE RATE / MONTH
C
C       XINSP ... INSPECTION TIME (1000 HOURS)
C
C       BETA .... ARRAY OF WEIBULL SLOPES FOR THE FAILURE MODES
C
C       ETA ..... ARRAY OF WEIBULL CHARACTERISTIC LIVES FOR THE FAILURE
C                 MODES
C
C       NMODE ... NUMBER OF FAILURE MODES
C
C       END ..... END CODE INDICATING END OF DATA
C
C
C *********************************************************************
C.....DIMENSION ARRAYS
C *********************************************************************
C
      CHARACTER*4 DUMMY,ANS,XNO,XNO1
      DIMENSION TIME(2000),BETA(15),ETA(15),CAL(121),XMODE(15)
C
      DATA XNO/'N   '/,XNO1/'n   '/
C
C
C *********************************************************************
C.....BEGIN ROUTINE
C *********************************************************************
C
C.....SET INPUT AND OUTPUT FILE DEFINITIONS
      OPEN(4,FILE='MONTE.DAT')
      INF2   = 4
      XINSP = 1000.
      NSIM = 10
C.....WRITE HEADER TO SCREEN
      WRITE(*,9010)
 9010 FORMAT(//,20X,' WELCOME TO THE WONDERFUL WORLD OF',/,
     &           25X,'MONTE CARLO SIMULATION',
     &        //,20X,' TYPE GO AND HIT ENTER TO CONTINUE')
      READ(*,1010) DUMMY
 1010 FORMAT(A4)
C     WRITE(*,9011)
C9011 FORMAT('1',//,20X,'ENTER YOUR POPULATION TIMES ONE AT A TIME,',/,
C    &        20X,'WITH A DECIMAL (I.E. 457.3)',/,
C    &        20X,'FOLLOWING EACH WITH A CARRIAGE RETURN',//,20X,
C    &        'TERMINATE THE INPUT WITH A NEGATIVE VALUE')
```

```
      WRITE(*,9112)
 9112 FORMAT(///,20X,'THE POPULATION TIMES AS ENTERED IN MONTE.DAT:',/)
      BTIME = 0.
      I = 0
   11 I = I + 1
      READ(INF2,*) TIME(I)
      TIME(I) = TIME(I) + .001
      WRITE(*,9012) TIME(I)
 9012 FORMAT(30X,'TIME = ',F10.2)
      IF (TIME(I) .GT. BTIME) BTIME = TIME(I)
      IF (TIME(I) .GE. 0.01) GO TO 11
      NPTS = I - 1
      WRITE(*,9013)
 9013 FORMAT(///,20X,'ENTER THE BETA AND ETA PARAMETERS FOR YOUR',/,
     &       20X,'WEIBULL FAILURE MODES AS PAIRS, ONE PER LINE',/,
     &       20X,'WITH A SPACE IN BETWEEN (I.E. 2.5 5098.)',//,
     &       20X,'TERMINATE THE INPUT WITH A PAIR OF NEGATIVE VALUES')
      I = 0
  222 I = I + 1
      READ(*,*) BETA(I),ETA(I)
      WRITE(*,9014) BETA(I),ETA(I)
 9014 FORMAT(///,5X,'BETA = ',F10.4,5X,'ETA = ',F10.3)
      IF (BETA(I) .GT. 0.0 .AND. ETA(I) .GT. 0.0) GO TO 222
      NMODE = I - 1
      WRITE(*,9015)
 9015 FORMAT(///,20X,'ENTER YOUR USAGE RATE PER MONTH WITH A DECIMAL')
      READ(*,*) USE
      WRITE(*,9016) USE
 9016 FORMAT(//,20X,'USAGE RATE = ',F10.1)
      WRITE(*,9035)
 9035 FORMAT(///,20X,'ENTER YOUR INSPECTION INTERVAL WITH A DECIMAL')
      READ(*,*) XINSP
      WRITE(*,9036) XINSP
 9036 FORMAT(//,20X,'INSPECTION INTERVAL = ',F10.1)
      WRITE(*,9017)
 9017 FORMAT(//,20X,'ENTER THE MONTH AND YEAR FOR THE START OF THE',
     &       ' RISK ANALYSIS',/,20X,'I.E. 5 1986')
      READ(*,*) MONTH,MYEAR
      WRITE(*,9018) MONTH,MYEAR
 9018 FORMAT(//,20X,'MONTH AND YEAR FOR RISK ANALYSIS ',I2,3X,I4)
      WRITE(*,9019)
 9019 FORMAT(//,20X,'ENTER THE DURATION OF THE RISK ANALYSIS IN YEARS')
      READ(*,*) MYRS
      WRITE(*,9020) MYRS
 9020 FORMAT(//,20X,'DURATION OF RISK ANALYSIS IN YEARS = ',I4)
      WRITE(*,9022)
 9022 FORMAT(//,20X,'ENTER THE RANDOM NUMBER SEED FOR THE SIMULATION',/,
     &       20X,'SEED MUST BE AN ODD INTEGER LESS THAN 30000',/,
     &       20X,'I. E. 13931  (NO DECIMAL)')
      READ(*,*) IX
      WRITE(*,9023) IX
 9023 FORMAT(//,20X,'RANDOM NUMBER SEED = ',I10)
      WRITE(*,9042)
 9042 FORMAT(//,20X,'ENTER THE NUMBER OF SIMULATION TRIALS',/,
     &       20X,'NUMBER OF SIMULATION TRIALS SHOULD BE AN INTEGER',/,
     &       20X,'LESS THAN OR EQUAL TO 100 (I.E. 50) NO DECIMAL')
      READ(*,*) NSIM
      WRITE(*,9043) NSIM
 9043 FORMAT(//,20X,'NUMBER OF SIMULATION TRIALS = ',I4)
 98   IY = 32355
```

```
      IZ = 21459                                                        MON01190
      DO 6 I = 1,121                                                    MON00480
    6 CAL(I) = 0.                                                       MON00490
      DO 66 I=1,15
   66 XMODE(I)=0.
      FTIME = FLOAT(MYRS)*12.*USE                                       MON01270
      YUSE = 12.*USE                                                    MON01280
      NFAIL = 0                                                         MON01290
      DO 1 I = 1,NSIM                                                   MON01300
         DO 2 J = 1,NPTS                                                MON01310
            ACCTIM = 0.                                                 MON01320
            INT = TIME(J)/XINSP                                         MON01330
            XTIME = TIME(J)                                             MON01340
            TUNIT = TIME(J) - INT*XINSP                                 MON01350
            TINSP = XINSP - TUNIT                                       MON01360
            NINSP = FTIME/XINSP + 1                                     MON01370
            DO 44 K = 1,NINSP                                           MON01380
   22          XFAIL = 99999.                                           MON01390
               DO 33 L = 1,NMODE                                        MON01400
                  CALL RANDU(IX,IY,IZ,P)                                MON01410
                  XT = ETA(L)*((ALOG(1./(1. - P)))**(1./BETA(L)))       MON01420
                  IF (XT .LT. XFAIL) ISMODE = L                         MON01430
                  IF (XT .LT. XFAIL) XFAIL = XT                         MON01440
   33          CONTINUE                                                 MON01450
               IF (XFAIL .GE. TINSP) GO TO 38                           MON01460
               TUNIT = TUNIT + XFAIL                                    MON01470
               TINSP = TINSP - XFAIL                                    MON01480
               XTIME = XTIME + XFAIL                                    MON01490
               ACCTIM = ACCTIM + XFAIL                                  MON01500
               IF (ACCTIM .GE. FTIME) GO TO 2                           MON01510
               INDEX = ACCTIM/YUSE + 1                                  MON01520
               CAL(INDEX) = CAL(INDEX) + 1.                             MON01530
               XMODE(ISMODE) = XMODE(ISMODE) + 1.                       MON01540
               NFAIL = NFAIL + 1                                        MON01550
               IF(NSIM .LE. 2)
     *         WRITE(*,3000)P,XT,ISMODE,XFAIL,TUNIT,TINSP,XTIME,ACCTIM
     *         ,INDEX,YUSE,INT,I,J,NFAIL
 3000    FORMAT(F15.8,F12.1,I3,4F12.1,/,2X,F12.1,I5,F12.4,4I5)
               GO TO 22                                                 MON01560
   38          TUNIT = 0.                                               MON01570
               XTIME = XTIME + TINSP                                    MON01580
               ACCTIM = ACCTIM + TINSP                                  MON01590
               TINSP = XINSP                                            MON01600
   44       CONTINUE                                                    MON01610
    2    CONTINUE                                                       MON01620
    1 CONTINUE                                                          MON01630
      DO 7 I = 1,MYRS                                                   MON01640
    7 CAL(I) = CAL(I)/FLOAT(NSIM)                                       MON01650
      DO 8 I = 1,NMODE                                                  MON01660
    8 XMODE(I) = XMODE(I)/FLOAT(NSIM)                                   MON01670
      WRITE(*,9025) NSIM                                                MON01680
 9025 FORMAT(35X,'AVERAGE # OF',/,35X,'INCIDENTS IN',I4,/,              MON01690
     &       2X,'1ST DAY OF THROUGH LAST DAY OF',3X,'SIMULATION TRIALS')MON01700
      MO = MONTH - 1                                                    MON01710
      IF (MO .EQ. 0) MO = 12                                            MON01720
      DO 9 I = 1,MYRS                                                   MON01730
         MY = MYEAR + I - 1                                             MON01740
         MYP1 = MY + 1                                                  MON01750
         IF (MONTH .EQ. 1) MYP1=MY
         WRITE(*,9021) MONTH,MY,MO,MYP1,CAL(I)                          MON01760
```

```
9021      FORMAT(5X,I2,'/',I4,' - ',I2,'/',I4,8X,F10.1)                          MON01770
   9 CONTINUE                                                                    MON01780
      DO 55 I = 1,NMODE                                                          MON01790
         WRITE(*,9051) I,XMODE(I)                                                MON01800
 9051    FORMAT(5X,'MODE ',I2,3X,'NUMBER OF INCIDENTS ',F10.1)                   MON01810
  55 CONTINUE                                                                    MON01820
C     WRITE(*,9085) NFAIL                                                        MON01830
C9085 FORMAT(' NFAIL = ',I10)                                                    MON01840
      WRITE(*,900)
 900  FORMAT(2X,'DO YOU WANT TO CHANGE SOME INPUT AND RERUN (Y OR N)?')
      READ(*,901)ANS
 901  FORMAT(A4)
      IF (ANS .EQ. XNO .OR. ANS .EQ. XNO1) GO TO 99
      CALL REDO(TIME,BETA,ETA,USE,XINSP,MONTH,YEAR,MYRS,IX,NMODE,XMODE,
     * NSIM)
      GO TO 98
 99   CLOSE(4,STATUS='KEEP')                                                     MON01850
      STOP                                                                       MON01860
      END                                                                        MON01870
      SUBROUTINE RANDU(IX,IY,IZ,P)                                               MON01880
      IX = 171*MOD(IX,177) - 2*(IX/177)                                          MON01890
      IY = 172*MOD(IY,176) - 35*(IY/176)                                         MON01900
      IZ = 170*MOD(IZ,178) - 63*(IZ/178)                                         MON01910
      IF (IX .LT. 0) IX = IX + 30269                                             MON01920
      IF (IY .LT. 0) IY = IY + 30307                                             MON01930
      IF (IZ .LT. 0) IZ = IZ + 30323                                             MON01940
      XXX = FLOAT(IX)/30269. + FLOAT(IY)/30307. + FLOAT(IZ)/30323.               MON01950
      P = AMOD(XXX,1.)                                                           MON01960
      RETURN                                                                     MON01970
      END                                                                        MON01980
      SUBROUTINE REDO(TIME,BETA,ETA,USE,XINSP,MONTH,YEAR,MYRS,IX
     * ,NMODE,XMODE,NSIM)
      DIMENSION TIME(2000),BETA(15),ETA(15),CAL(121),XMODE(15)                   MON00370
C                                                                                MON00380
      CHARACTER*4 DUMMY                                                          MON00390
 98   WRITE(*,100)
 100  FORMAT(2X,'WHICH OF THE INPUTS WOULD YOU LIKE TO CHANGE?',/,
     1        2X,' 1. BETA AND/OR ETA',/,
     2        2X,' 2. USAGE RATE',/,
     3        2X,' 3. INSPECTION INTERVAL',/,
     4        2X,' 4. MONTH/YEAR FOR SIMULATION START',/,
     5        2X,' 5. DURATION OF RISK ANALYSIS (IN YEARS)',/,
     6        2X,' 6. RANDOM NUMBER SEED',/,
     7        2X,' 7. NUMBER OF SIMULATION TRIALS',/,
     8        2X,' 8. NO FURTHER CHANGES',/,
     9        2X,' (ENTER A NUMBER FROM 1 TO 8)',/)
      READ(*,101)NUMB
 101  FORMAT(I1)
      GO TO (10,20,30,40,50,60,70,80),NUMB
 10   WRITE(*,8013)                                                              MON00770
 8013 FORMAT(//,20X,'THE CURRENT BETA S AND ETA S  IN THE ',/,                   MON00780
     &        20X,'SIMULATOR ARE:')
      DO 8014 I=1,NMODE
 8014 WRITE(*,8015)BETA(I),ETA(I)
 8015 FORMAT(/,20X,F10.4,5X,F10.4)
      WRITE(*,9013)                                                              MON00770
 9013 FORMAT(///,20X,'ENTER THE BETA AND ETA PARAMETERS FOR YOUR',/,             MON00780
     &        20X,'WEIBULL FAILURE MODES AS PAIRS, ONE PER LINE',/,              MON00790
     &        20X,'WITH A SPACE IN BETWEEN (I.E. 2.5 5098.)',//,                 MON00800
     &        20X,'TERMINATE THE INPUT WITH A PAIR OF NEGATIVE VALUES')          MON00810
```

```
      I = 0                                                             MONU0820
  222 I = I + 1                                                         MON00830
      READ(*,*) BETA(I),ETA(I)                                          MON00840
      WRITE(*,9014) BETA(I),ETA(I)                                      MON00850
 9014 FORMAT(///,5X,'BETA = ',F10.4,5X,'ETA = ',F10.3)                  MON00860
      IF (BETA(I) .GT. 0.0 .AND. ETA(I) .GT. 0.0) GO TO 222             MON00880
      NMODE = I - 1                                                     MON00890
      GO TO 98
 20   WRITE(*,9015)USE                                                  MON00900
 9015 FORMAT(///,20X,'YOUR CURRENT USAGE RATE PER MONTH IS  ',F10.1,    MON00910
     *//,20X,'ENTER YOUR NEW USAGE RATE PER MONTH WITH A DECIMAL')
      READ(*,*) USE                                                     MON00920
      WRITE(*,9016) USE                                                 MON00930
 9016 FORMAT(//,20X,'USAGE RATE = ',F10.1)                              MON00940
      GO TO 98
 30   WRITE(*,9035)XINSP                                                MON00950
 9035 FORMAT(///,20X,'YOUR CURRENT INSPECTION INTERVAL IS  ',F10.1,     MON00960
     1//,20X,'ENTER YOUR NEW INSPECTION INTERVAL WITH A DECIMAL')
      READ(*,*) XINSP                                                   MON00970
      WRITE(*,9036) XINSP                                               MON00980
 9036 FORMAT(//,20X,'INSPECTION INTERVAL = ',F10.1)                     MON00990
      GO TO 98
 40   WRITE(*,9017)                                                     MON01000
 9017 FORMAT(//,20X,'ENTER THE MONTH AND YEAR FOR THE START OF THE',    MON01010
     &       ' RISK ANALYSIS',/,20X,'I.E. 5 1986')                      MON01020
      READ(*,*) MONTH,MYEAR                                             MON01030
      WRITE(*,9018) MONTH,MYEAR                                         MON01040
 9018 FORMAT(//,20X,'MONTH AND YEAR FOR RISK ANALYSIS ',I2,3X,I4)       MON01050
      GO TO 98
 50   WRITE(*,9019)MYRS                                                 MON01060
 9019 FORMAT(//,20X,'THE CURRENT RISK ANALYSIS DURATION IS  ',I4,       MON01070
     1//,20X,'ENTER THE NEW DURATION OF THE RISK ANALYSIS IN YEARS')
      READ(*,*) MYRS                                                    MON01080
      WRITE(*,9020) MYRS                                                MON01090
 9020 FORMAT(//,20X,'DURATION OF RISK ANALYSIS IN YEARS = ',I4)         MON01100
      GO TO 98
 60   WRITE(*,9022)IX                                                   MON01110
 9022 FORMAT(//,20X,'YOUR CURRENT RANDOM NUMBER SEED IS  ',I12,         MON01120
     1//,20X,'ENTER THE NEW RANDOM NUMBER SEED FOR THE SIMULATION',/,
     &       20X,'SEED MUST BE AN ODD INTEGER LESS THAN 30000',/,       MON01130
     &       20X,'I. E. 13931  (NO DECIMAL)')                           MON01140
      READ(*,*) IX                                                      MON01150
      WRITE(*,9023) IX                                                  MON01160
 9023 FORMAT(//,20X,'RANDOM NUMBER SEED = ',I10)                        MON01170
      GO TO 98
 70   WRITE(*,9042)NSIM                                                 MON01200
 9042 FORMAT(//,20X,'THE CURRENT NUMBER OF SIMULATION TRIALS IS   ',I3, MON01210
     1//,20X,'ENTER THE NEW NUMBER OF SIMULATION TRIALS',/,
     &       20X,'NUMBER OF SIMULATION TRIALS SHOULD BE AN INTEGER',/,  MON01220
     &       20X,'LESS THAN OR EQUAL TO 100 (I.E. 50) NO DECIMAL')      MON01230
      READ(*,*) NSIM                                                    MON01240
      WRITE(*,9043) NSIM                                                MON01250
 9043 FORMAT(//,20X,'NUMBER OF SIMULATION TRIALS = ',I4)                MON01260
      GO TO 98
 80   RETURN
      END
```

3. Random Number Generator Subroutine

A subroutine for generating random numbers used in the Monte Carlo Simulation Program.

Input is a random number seed that must be an odd integer less than 30000.

RANDOM NUMBER GENERATOR SUBROUTINE

```
      DIMENSION TIME(2000),BETA(15),ETA(15),CAL(121),XMODE(15)          MON00370
      READ(*,*) IX                                                      MON01150
      WRITE(*,9023) IX                                                  MON01160
 9023 FORMAT(//,20X,'RANDOM NUMBER SEED = ',I10)                        MON01170
      OPEN(4,FILE='A:MONTED.DAT',STATUS='NEW')                          MON00470
      IY = 32355                                                        MON01180
      IZ = 21459                                                        MON01190
      NSIM=1000
      DO 33 L = 1,1000                                                  MON01400
                CALL RANDU(IX,IY,IZ,P)                                  MON01410
      TIME(L)=P
C               XT = ETA(L)*((ALOG(1./(1. - P)))**(1./BETA(L)))         MON01420
   33        CONTINUE                                                   MON01450
      DO 34 I=1,NSIM
   34  WRITE(4,100)TIME(I)
  100 FORMAT(F10.7)
      CALL HIST(TIME,NSIM)                                              MON01660
      STOP                                                              MON01860
      END                                                               MON01870
      SUBROUTINE RANDU(IX,IY,IZ,P)                                      MON01880
      IX = 171*MOD(IX,177) - 2*(IX/177)                                 MON01890
      IY = 172*MOD(IY,176) - 35*(IY/176)                                MON01900
      IZ = 170*MOD(IZ,178) - 63*(IZ/178)                                MON01910
      IF (IX .LT. 0) IX = IX + 30269                                    MON01920
      IF (IY .LT. 0) IY = IY + 30307                                    MON01930
      IF (IZ .LT. 0) IZ = IZ + 30323                                    MON01940
      XXX = FLOAT(IX)/30269. + FLOAT(IY)/30307. + FLOAT(IZ)/30323.      MON01950
      P = AMOD(XXX,1.)                                                  MON01960
      RETURN                                                            MON01970
      END                                                               MON01980
      SUBROUTINE HIST(PCT,N)
      DIMENSION PCT(1),H(20),NN(20)
      XN=N
      WRITE(*,100)
  100 FORMAT(10X,'HISTOGRAM INTERVAL IS',18X,'NO.',/12X,
     * 'FROM',20X,'TO',11X,'PTS')
      NINT=1.5+3.3*ALOG(XN)*.4342945
      NINT1=NINT+1
      SM=9999999.
      BIG=-9999999.
      DO 250 I=1,N
      IF (PCT(I)-SM)248,249,249
  248  SM=PCT(I)
  249 IF (PCT(I)-BIG)250,247,247
  247 BIG=PCT(I)
  250 CONTINUE
      R=(BIG-SM)/FLOAT(NINT)
      DO 251 I=1,NINT1
      NN(I)=0
  251 H(I)=(I-1)*R+SM
      DO 252 I=1,N
      J=(PCT(I)-SM)/R+1
  252 NN(J)=NN(J)+1
      NN(NINT)=NN(NINT)+NN(NINT1)
      DO 260 I=2,NINT1
  260 WRITE(*,1020)H(I-1),H(I),NN(I-1)
 1020 FORMAT(2E20.7,I10)
      RETURN
      END
```

VITA

Woodrow T. Roberts, Jr., is the Superintendent of the Plastics Central Maintenance Department of Dow Chemical Company, Louisiana Division in Plaquemine, Louisiana. Mr. Roberts received a Bachelor of Science degree in Chemical Engineering from Auburn University in 1966 and a Masters degree in Business Administration from Louisiana State University in 1974.

He has been with Dow Chemical since 1966 and has worked in several areas of plant operations including being Superintendent of the Louisiana Division Polyethylene A plant and Superintendent of the Cellulose Ethers plant prior to his present position.

Mr. Roberts has served as President of the Baton Rouge Chapter of the Society of Reliability Engineers (SRE) and is presently the Chapter's representative to the International SRE Executive Board of Directors.

The author presented a paper entitled "Reliability-Centered Maintenance" at the Dow Louisiana Technology Exchange Conference (LATEC) in March, 1987. He gave the opening remarks entitled, "Quality and Reliability, a Continuous Improvement Process" at the Annual Conference of the Baton Rouge Chapter of the SRE on March 6, 1990.

Mr. Roberts assisted Dr. Robert Abernethy in the presentation of a "Weibull Reliability, Maintainability and Failure Analysis Workshop, " March 7, 1990 in conjunction with the SRE conference.

The author presented a paper entitled "How to Introduce a Reliability-Centered Maintenance Program" at the Houston Round Table's Quality Day on September 27, 1990.

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Candidate: Woodrow T. Roberts, Jr.

Major Field: Engineering Science

Title of Dissertation: Failure Predictions in Repairable Multi-Component Systems

Approved:

L. Mann

Major Professor and Chairman

Daniel Fogel

Dean of the Graduate School

EXAMINING COMMITTEE:

John R. Collier

Armando B. Corripio

T. Frank R

V. Aghazadeh

James M Pruett

Ralph W. Pike

Date of Examination:

18 Mar 1992